Algebra 3: Revision Exercise Core

Revision Exercise (Core)

Q1 $\quad -1 \leq \frac{2x+4}{3} \leq 2$

mult by 3 $\quad -3 \leq 2x+4 \leq 6$

minus 4 $\quad -7 \leq 2x \leq 2$

$\div 2$ $\quad -7/2 \leq x \leq 1$

Q2 (a) (i) $10^{3.5} = 3162.278$

(ii) $\log_{10} 4.5 = 0.6532$

(iii) $10^{3(0.04)} = 10^{0.12} = 1.32$

(iv) $\log 5(100) = \log 500 = 2.69897$

(b) (i) $e^{3.4} = 29.96$ $\quad$ Alpha key or $\times 10^x$

(ii) $\ln 589 = 6.378$

(iii) $e^{-0.02(40)-4} = e^{-4.8} = 8.2297 \times 10^{-3} = 0.00823$

(iv) $\ln\left(\frac{10}{3.7}\right) = 0.9943$

Q3 (i) $\quad f(x) = 3 \times 4^x$

$(a, 6) \Rightarrow 3 \times 4^a = 6$

$4^a = 2$

$2^{2a} = 2^1$

$2a = 1$

$a = 1/2$

(ii) $\left(-\frac{1}{2}, b\right) \Rightarrow 3 \times 4^{-1/2} = b$

$3 \times \frac{1}{\sqrt{4}} = b$

$\frac{3}{2} = b$

Q4 $|x-8| = 3$

$x-8 = 3$
$x = 11$

$x-8 = -3$
$x = 5$

Q5 (i) $5^{2n} \times 25^{2n-1} = 625$

$5^{2n} \times 5^{2(2n-1)} = 5^4$

$5^{2n+2(2n-1)} = 5^4$

$2n + 2(2n-1) = 4$

$2n + 4n - 2 = 4$

$6n = 6$

$n = 1$

(ii) $27^{n-2} = 9^{3n+2}$

$3^{3(n-2)} = 3^{2(3n+2)}$

$3n - 6 = 6n + 4$

$-10 = 3n$

$-10/3 = n$

Q6 $y = a2^x + b$

(i)

$(0, 2.5) \Rightarrow 2.5 = a2^0 + b$ $\quad [2^0 = 1]$

$2.5 = a + b$

$(2, 4) \Rightarrow 4 = a2^2 + b$

$4 = 4a + b$

(ii)

$-a - b = -2.5$

$4a + b = 4$

$3a = 1.5$

$\underline{a = 0.5}$

$a + b = 2.5$

$0.5 + b = 2.5$

$\underline{b = 2}$

Q7

(i) $\ln(x)$ = Graph C [through (1,0)]

(ii) $\ln(x+1)$ = Graph B [shifts 1 to left]

(iii) $\ln(x)+1$ = Graph A [shift graph up 1]

Q8

$$\ln(x-1)+\ln(x+2)=\ln(6x-8)$$
$$\ln(x-1)(x+2)=\ln(6x-8)$$
$$(x-1)(x+2)=6x-8$$
$$x^2+x-2=6x-8$$
$$x^2-5x+6=0$$
$$(x-3)(x-2)=0$$
$$x=3 \qquad x=2.$$

Q9

$y=Ae^{bt}$ when $y=6$, $t=1$

① $\boxed{6=Ae^{b}}$

when $y=8$, $t=2$ ② $\boxed{8=Ae^{2b}}$

to find b. from ① $A=\frac{6}{e^b}$ and from ② $A=8/e^{2b}$

equate: $\frac{6}{e^b}=\frac{8}{e^{2b}}$

$$\frac{e^{2b}}{e^b}=\frac{8}{6}$$
$$e^b=4/3$$
$$\ln e^b=\ln 4/3$$
$$b\ln e=\ln 4/3$$
$$b(1)=\ln 4/3$$
$$b=0.288$$

find A. $A=\frac{6}{e^b}$

$A=\frac{6}{e^{\ln(4/3)}}=\frac{9}{2}$ (calculator)

to find A

from ① $e^b=\frac{6}{A}$ from ② $e^{2b}=\frac{8}{A}$

sub in

$$(e^b)^2=8/A$$
$$\left(\frac{6}{A}\right)^2=8/A$$
$$36/A^2=8/A$$
$$36/8=A^2/A$$
$$\frac{9}{2}=A.$$

find b $e^b=\frac{6}{9/2}$

$e^b=4/3$

$\ln e^b=\ln 4/3$ $b=\ln 4/3$

$b\ln e=\ln 4/3$

Q10 $\quad y = a\log_2(x-b)$

$(5,2)$ $\quad 2 = a\log_2(5-b)$

$\Rightarrow 2 = \log_2(5-b)^a$

$2^2 = (5-b)^a$

$\boxed{4 = (5-b)^a}$

$(7,4)$ $\quad 4 = a\log_2(7-b)$

$4 = \log_2(7-b)^a$

$2^4 = (7-b)^a$

$\boxed{16 = (7-b)^a}$

Write each in terms of $b = \square$ and equate

$4 = (5-b)^a$ $\qquad$ $16 = (7-b)^a$

$4^{1/a} = 5-b$ $\qquad$ $16^{1/a} = 7-b$

$4^{1/a} - 5 = -b$ $\qquad$ $16^{1/a} - 7 = -b$

$b = 5 - 4^{1/a}$ $\qquad$ $b = 7 - 16^{1/a}$

$5 - 4^{1/a} = 7 - 16^{1/a}$ $\quad$ (Make base N°s Same)

$5 - 4^{1/a} = 7 - 4^{2(1/a)}$

$4^{2(1/a)} - 4^{1/a} - 2 = 0$

Let $y = 4^{1/a}$ $\quad y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2 \qquad y = -1$

sub back in $\quad 4^{1/a} = 2 \qquad 4^{1/a} = -1$ (not valid)

$2^{2(1/a)} = 2^1$

$\frac{2}{a} = 1$

$\boxed{2 = a}$

$b = 5 - 4^{1/a} \Rightarrow b = 5 - 4^{1/2} \Rightarrow b = 5 - 2 \Rightarrow \boxed{b = 3}$

Q11 $\quad 32^{x-1} = 28$ $\quad$ (cannot get base N's the same $\Rightarrow$ Use ln)

$$\ln 32^{(x-1)} = \ln 28$$
$$(x-1)\ln 32 = \ln 28$$
$$x-1 = \frac{\ln 28}{\ln 32}$$
$$x-1 = \ln\left(\frac{28}{32}\right)$$
$$x-1 = 0.96147$$
$$x = 1.96147$$
$$x = 1.96.$$

Q12

$$3+6+9+12+ \quad 3n = \frac{3n}{2}(n+1) \qquad n \geq 1$$

Show true for $n=1$ $\quad 3 = \frac{3(1)}{2}(1+1) \Rightarrow 3 = 3$ True

Assume true for $n=k$

$$3+6+9+12+\cdots\ 3k = \frac{3k}{2}(k+1)$$

Prove true for $n=k+1$

$$3+6+9+12+\cdots 3(k+1) = \frac{3(k+1)}{2}((k+1)+1)$$

add $(k+1)$ to both sides of original $\qquad$ need to show [illegible]

$$3+6+9+12+\cdots 3k + 3(k+1) = \frac{3k}{2}(k+1) + 3(k+1)$$
$$= \frac{3k(k+1) + (2)(3)(k+1)}{2}$$
$$= \frac{[3(k+1)](k+2)}{2}$$
$$= \frac{3(k+1)}{2}((k+1)+1) \quad \Rightarrow \text{true for } n=k+1$$

Since true for $n=1$, $\Rightarrow$ true for $n = 1+1 = 2$

Since true for $n=2$, $\Rightarrow$ true for $n = 2+1 = 3$ etc

$\therefore$ true for all values of n, $n \geq 1$.

Q13 $8^n + 6$ is div by 7

Show true for $n=1$, $8^1+6 = 14$, is div by 7
Assume true for $n=k$ $\Rightarrow$ 8^k+6 is div by 7
Show true for $n=k+1$
$\Rightarrow$ prove: $8^{k+1}+6$ is div by 7

$$8^k(8)+6$$
$$8^k(7+1)+6$$
$$\underbrace{8^k(7)}_{\text{div by } 7} + \underbrace{8^k+6}_{\text{div by } 7}$$

$\Rightarrow$ true for $n = k+1$

Since true for $n=1$, $\Rightarrow$ true for $n=1+1=2$
Since true for $n=2$ $\Rightarrow$ true for $n=2+1=3$ etc
$\therefore$ true for all values of n, $n \in N$

Q1b $\quad n^2 > 4n+3 \quad n \geq 5$

Show true for $n=5$, $5^2 > 4(5)+3$, $25 > 23$, True

Assume true for $n=k \Rightarrow$
$k^2 > 4k+3$

Prove true for $n=k+1 \Rightarrow$ prove $(k+1)^2 > 4(k+1)+3$.

$$(k+1)^2 = k^2+2k+1$$
$$(\text{But } k^2 > 4k+3) \rightarrow \text{Sub in for } k^2$$

$$(k+1)^2 > 4k+3 + 2k+1$$
$$4k+4 + 2k$$
$$4(k+1) + 2k \qquad \text{but } 2k > 3 \text{ for } k \geq 5$$
$$\Rightarrow (k+1)^2 > 4(k+1)+3$$

Since true for $n=5 \Rightarrow$ true for $n=5+1=6$.
Since true for $n=6$, $\Rightarrow$ true for $n=6+1=7$ etc
$\therefore$ true for all values of n, $n \geq 5$, $n \in \mathbb{N}$