Revision Exercise (Extended-Response Questions)

Q1 $N = 5000e^{-0.15t}$

(i) $t = 0 \Rightarrow N = 5000e^{-0.15(0)} = 5000e^{0} = 5000(1) = 5000$

$t = 5 \Rightarrow N = 5000e^{-0.15(5)} = 5000e^{-0.75} = 5000(0.47236655)$

$= 2361.83 = 2362.$

$\Rightarrow$ claim is justified

(ii) $t = 10 \Rightarrow N = 5000e^{-0.15(10)} = 5000e^{-1.5}$

$= 5000(0.23313)$

$= 1115.65$

(iii) $t = 0 \Rightarrow N = 5000e^{-0.15(0)} = 5000e^{0} = 5000(1) = 5000$

(iv) $N = 100$

$100 = 5000e^{-0.15(t)}$

$\frac{100}{5000} = e^{-0.15t}$

$0.02 = e^{-0.15t}$

$\ln 0.02 = -0.15t \ln e$

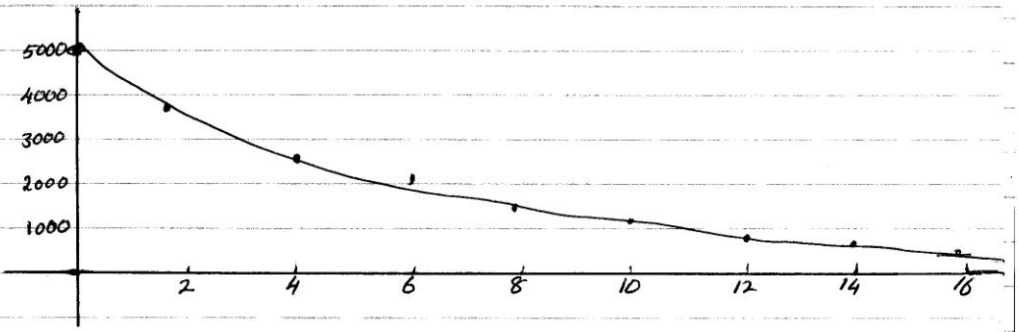
$-3.912 = -0.15t(1)$

$\frac{3.912}{0.15} = t$

$26.08 = t$ 26.08 days

(v)

t	0	2	4	6	8	10	12	14	16
$N = 5000e^{-0.15t}$	5000	3704	2744	2033	1506	1116	826	612	454



Q2

(i) $A = 0.02(0.92)^{x/10}$

(ii) $\frac{1}{3}$ length $= \frac{5}{3}m = 1.667$

$$A = 0.02(0.92)^{\frac{1.667}{10}}$$

$$= 0.02(0.92)^{0.1667}$$

$$= 0.019724 = 0.0197.$$

(iii) $S = (0.92)^{10-3x}$

$$W = S \times C$$

$$W = (0.92)^{10-3x}(0.02)(0.92)^{x/10}$$

$$= (0.02)(0.92)^{10-3x+x/10}$$

$$= (0.02)(0.92)^{10-2.9x}$$

$$\left[\begin{array}{l} 10-3x+\frac{x}{10} \\ 10-3x+0.1x \\ 10-2.9x \end{array}\right]$$

(iv) $W < 0.02 \times (0.92)^{2.5}$

$$\cancel{0.02}(0.92)^{10-2.9x} < \cancel{0.02}(0.92)^{2.5}$$

$$10-2.9x < 2.5$$

$$-2.9x < 2.5-10$$

$$-2.9x < -7.5$$

change signs $\quad 2.9x > 7.5$

$$x > 7.5/2.9$$

$$x > 2.586$$

$$x > 2.59$$

Q3

(i) ~~A~~ 17% decrease $\Rightarrow$ 83% Left. $= 0.83$

Intensity of $A = (.83)^n I$

B 11% decrease $\Rightarrow$ 89% Left $= 0.89$

Intensity of $B = (0.89)^n (0.66) I$

(ii)
$$A = B$$
$$(0.83)^n \cancel{I} = (0.66)(0.89)^n \cancel{I}$$
$$\log (0.83)^n = \log (0.66)(0.89)^n$$
$$\log (0.83)^n = \log (0.66) + \log (0.89)^n$$
$$n \log (0.83) = (-0.180456) + n \log (0.89)$$
$$n \log (0.83) - n \log (0.89) = -0.180456$$
$$n (\log 0.83 - \log 0.89) = -0.180456$$
$$n (-0.0303119) = -0.180456$$
$$n = \frac{-0.180456}{-0.0303119}$$
$$n = 5.9533$$
$n = 6$ ~~days~~ stations

Q4

~~(B)~~ reducing = depreciation increasing ⇒ Compound Int.

(i) Grey ⇒ Compound Int formula.

$$P = A(1+0.11)^t = A(1.11)^t$$

(ii) red ⇒ depreciation formula.

$$P = (10)A(1-0.05)^t = (10)A(0.95)^t$$

(iii) $\cancel{A}(1.11)^t = 10\cancel{A}(0.95)^t$

$$\left(\frac{1.11}{0.95}\right)^t = 10$$

$$(1.168421)^t = 10$$

$$\log(1.168421)^t = \log 10$$

$$t\log(1.168421) = 1$$

$$t = \frac{1}{\log(1.168421)} = 14.793 = 14.8 \text{ years}$$

(iv) ~~Proportions reversed ⇒ 10 times as many grey as red~~ ?

~~⇒ Red would have to decrease 10 times more.~~

$$\cancel{\Rightarrow (100)(A)(0.95)^t}$$

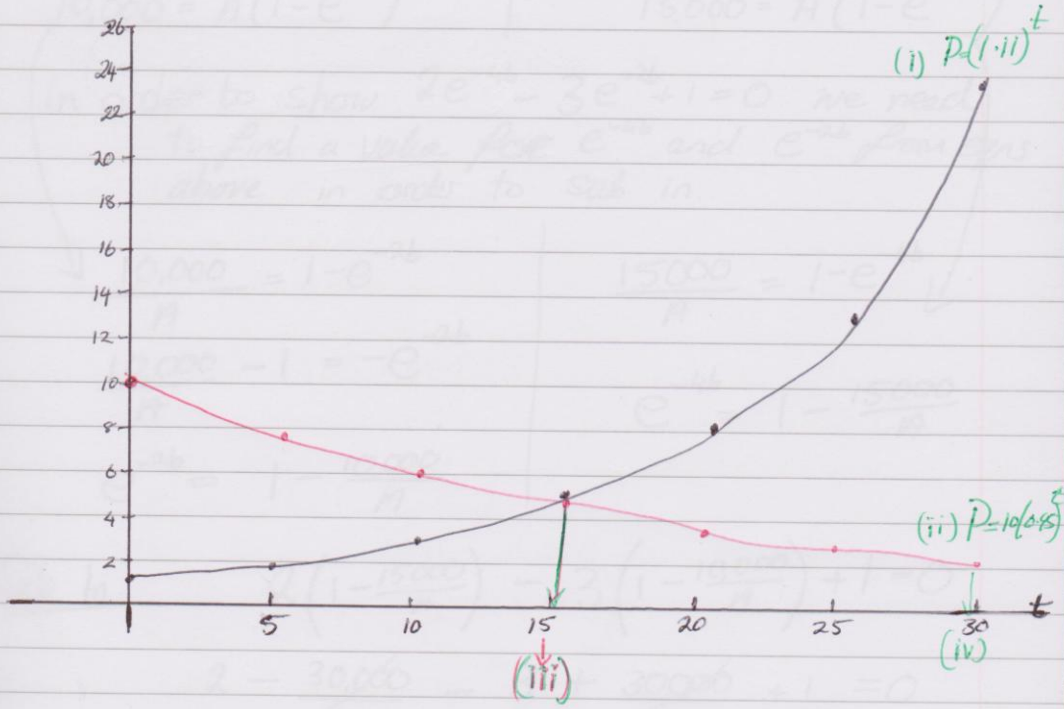
(iv) Proportions reversed $\Rightarrow$ 10 times as many red as grey?
$\Rightarrow$ Red would have to decrease 10 times more.
$\Rightarrow$ $10 \times 10A(0 \cdot 95)^t = 100A(0 \cdot 95)^t$

$$\cancel{A}(1 \cdot 11)^t = 100\cancel{A}(0 \cdot 95)^t$$
$$\left(\frac{1 \cdot 11}{0 \cdot 95}\right)^t = 100$$
$$(1 \cdot 168421)^t = 100$$
$$\log(1 \cdot 168421)^t = \log 100$$
$$t \log(1 \cdot 168421) = 2.$$
$$t = \frac{2}{\log(1 \cdot 168421)} = 29 \cdot 586$$
$$t = 29 \cdot 6 \text{ years.}$$

(v)

t	$P=(1.11)^t$
0	1
5	1.7
10	2.8
15	4.8
20	8.1
25	13.6
30	23.9

t	$P=10(0.95)^t$
0	10
5	7.7
10	6.0
15	4.6
20	3.6
25	2.8
30	2.1



Q5 $\quad n = A(1-e^{-bt})$

(i) Growth: as t increases, n also increases
[$e^{\text{neg power}}$ is a fraction, as t increases the fraction gets smaller]

(ii) $t=2 \quad n=10{,}000$ | $t=4 \quad n=15{,}000$

$10{,}000 = A(1-e^{-2b})$ | $15000 = A(1-e^{-4b})$

In order to show $2e^{-4b} - 3e^{-2b} + 1 = 0$ we need to find a value for e^{-4b} and e^{-2b} from eqns above in order to sub in.

$$\frac{10{,}000}{A} = 1 - e^{-2b}$$

$$\frac{10{,}000}{A} - 1 = -e^{-2b}$$

$$e^{-2b} = 1 - \frac{10{,}000}{A}$$

$$\frac{15000}{A} = 1 - e^{-4b}$$

$$e^{-4b} = 1 - \frac{15000}{A}$$

Sub In $\quad 2\left(1 - \frac{15000}{A}\right) - 3\left(1 - \frac{10{,}000}{A}\right) + 1 = 0$

$$2 - \cancel{\frac{30{,}000}{A}} - 3 + \cancel{\frac{30000}{A}} + 1 = 0$$

$$2 - 3 + 1 = 0$$

$$0 = 0.$$

(iii) $2e^{-4b} - 3e^{-2b} + 1 = 0$

Let $a = e^{-2b}$ $\Rightarrow e^{-4b} = (e^{-2b})^2 = a^2$.

$\Rightarrow 2a^2 - 3a + 1 = 0$.

(iv) $(2a - 1)(a - 1) = 0$

$2a = 1$

$a = 1/2$ $\quad a = 1$

(v) $e^{-2b} = \frac{1}{2}$ or $e^{-2b} = 1$

$\ln e^{-2b} = \ln \frac{1}{2}$

$-2b \ln e = \ln 1 - \ln 2$

$-2b(1) = 0 - \ln 2$.

$2b = \ln 2$

$b = \frac{1}{2} \ln 2$.

$b = 0.3465735903$.

(vi) $n = A(1 - e^{-bt})$

when $t = 2$ $\quad n = 10{,}000$ $\quad b = \frac{1}{2}\ln 2$.

$10{,}000 = A\left(1 - e^{-\cancel{2}(\frac{1}{\cancel{2}}\ln 2)}\right)$

$10{,}000 = A\left(1 - e^{-\ln 2}\right)$

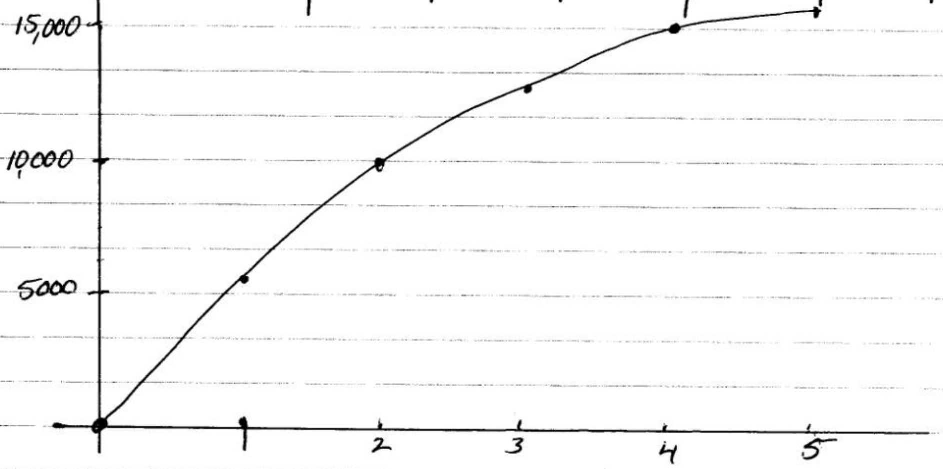
$10{,}000 = A\left(1 - \frac{1}{2}\right)$

$10\,000 = A\left(\frac{1}{2}\right)$

$20000 = A$.

(vii) $n = 20000\left(1 - e^{-t\left(\frac{1}{2}\ln 2\right)}\right)$

$t =$	0	1	2	3	4	5
$n =$	0	5858	10,000	12928	15000	16464



(viii)

$$18,000 = 20,000\left(1 - e^{-t\left(\frac{1}{2}\ln 2\right)}\right)$$

$$\frac{18,000}{20,000} = 1 - e^{-t\left(\frac{1}{2}\ln 2\right)}$$

$$\frac{9}{10} = 1 - e^{-t\left(\frac{1}{2}\ln 2\right)}$$

$$e^{-t\left(\frac{1}{2}\ln 2\right)} = 1 - \frac{9}{10}$$

$$e^{-t\left(\frac{1}{2}\ln 2\right)} = 0.1$$

$$-t\left(\tfrac{1}{2}\ln 2\right)\ln e = \ln 0.1$$

$$-t(0.3465735903)(1) = -2.302585093.$$

$$t(0.3465735903) = 2.302585093.$$

$$t = \frac{2.302585093}{0.3465735903}$$

$$t = 6.643850195 = 6.64 \text{ hrs.}$$