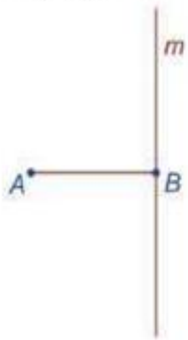
Constructing $\sqrt{2}$ and $\sqrt{3}$

$\sqrt{2}$ and $\sqrt{3}$ cannot be written as fractions, but can be constructed.

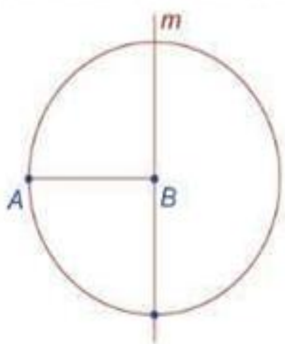
Construct $\sqrt{2}$

1. Let the line segment AB be of length 1 unit.

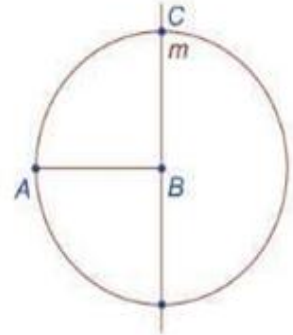
2. Construct a line m perpendicular to $[AB]$ at B.



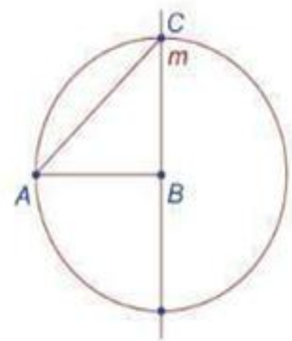
3. Construct a circle with centre B and radius $[AB]$.



4. Mark the intersection, C, of the circle and m.



5. Draw the line segment CA.
   $|AC| = \sqrt{2}$



Proof: $|AB| = |BC| = 1$ (radii of circle)

$|AB|^2 + |BC|^2 = |AC|^2$ (Theorem of Pythag

$1^2 + 1^2 = |AC|^2$

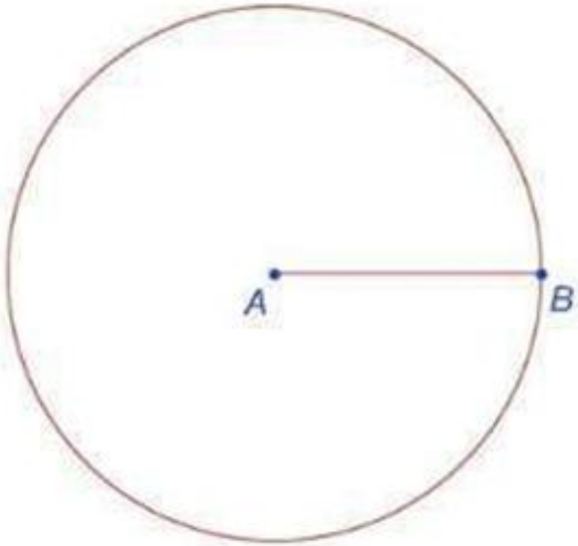
$|AC|^2 = 2$

$\therefore |AC| = \sqrt{2}$

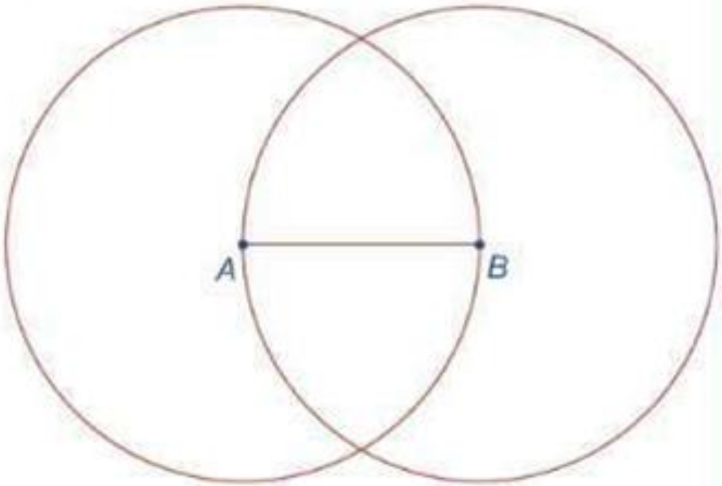
Construct $\sqrt{3}$

1. Let the line segment AB be of length 1 unit.

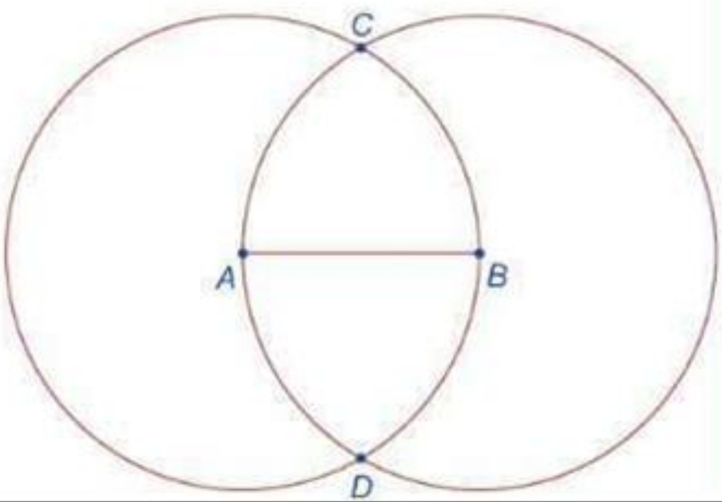
2. Construct a circle with centre A and radius length $|AB|$.



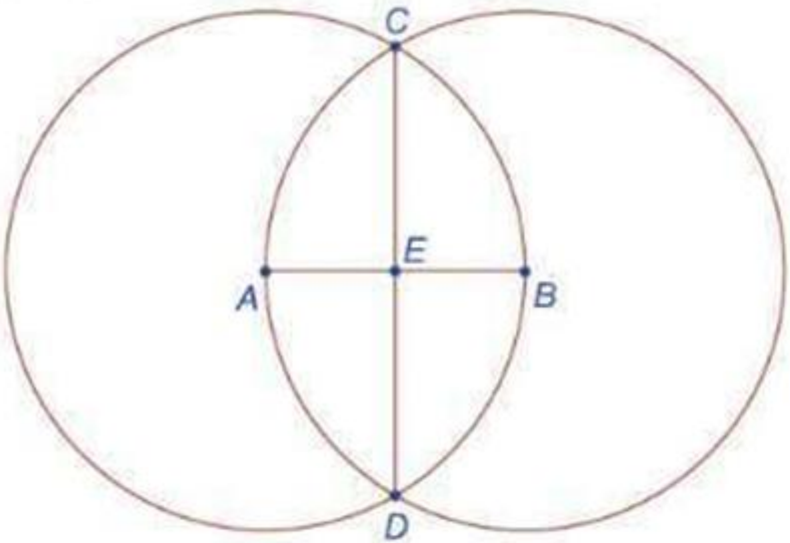
3. Construct a circle with centre B and radius length $|AB|$.



4. Mark the intersection of the two circles as C and D.



5. Draw the line segment [CD].
$|CD| = \sqrt{3}$



Proof: CD is the perpendicular bisector of [AB] (Construction).

$\therefore |AE| = |EB| = \frac{1}{2}$

$|AC| = |BC| = 1$ (Construction)

$|AE|^2 + |EC|^2 = |AC|^2$ (Theorem of Pythagoras)

$\left(\frac{1}{2}\right)^2 + |EC|^2 = 1^2$

$|EC|^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore |EC| = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$|CD| = 2\,|EC|$

$= 2\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow |CD| = \sqrt{3}$