Ex 4.1

Q1 (i) $\int x\,dx = \frac{x^2}{2} + C$

(iv) $\int -2x^2\,dx = \frac{-2x^3}{3} + C$

(vii) $\int (4x^3+6x)\,dx = \frac{\cancel{4}x^4}{\cancel{4}} + \frac{\cancel{6}^{3}x^2}{\cancel{2}} + C$

$= x^4 + 3x^2 + C$

Q2 (i) $\int x^{-2}\,dx = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$

(iv) $\int -\frac{2}{x^3}\,dx = \int -2x^{-3} = \frac{-\cancel{2}x^{-2}}{-\cancel{2}} + C = \frac{1}{x^2} + C$

(vii) $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x} + C$

Q3 (iv) $\int \sqrt{x} + \frac{1}{\sqrt{x}} = \int x^{\frac{1}{2}} + x^{-\frac{1}{2}} = \frac{x^{3/2}}{3/2} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{2}{3} x^{3/2} + 2x^{\frac{1}{2}} + C$

$= \frac{2}{3}\sqrt{x^3} + 2\sqrt{x} + C.$

(v) $\int \left(2\sqrt{x} - \frac{2}{x^2}\right) dx = \int \left(2x^{\frac{1}{2}} - 2x^{-2}\right) dx$

$= \frac{2x^{3/2}}{3/2} - \frac{2x^{-1}}{-1} + C$

$= \frac{4}{3}\sqrt{x^3} + \frac{2}{x} + C$

(vi) $\int \left(\frac{1}{x^2} - \frac{x}{\sqrt{x}}\right) dx = \int x^{-2} - x^{\frac{1}{2}}$

$= \frac{x^{-1}}{-1} - \frac{x^{3/2}}{3/2} + C$

$= -\frac{1}{x} - \frac{2}{3}\sqrt{x^3} + C$

Q4 (ii) $\frac{dy}{dx} = 6x^3 - 4x^2 + x - 5$

$y = \int 6x^3 - 4x^2 + x - 5 = \frac{6x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} - 5x + C$

$= \frac{3}{2}x^4 - \frac{4}{3}x^3 + \frac{x^2}{2} - 5x + C$

Q5 (iii) $\int \sqrt{x}(x-3)\,dx = \int x^{\frac{1}{2}}(x-3) = \int x^{3/2} - 3^{\frac{1}{2}}$

$= \frac{x^{5/2}}{5/2} - \frac{x^{3/2}}{3/2} + C = \frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$

$= \frac{2}{5}\sqrt{x^5} - \frac{2}{3}\sqrt{x^3} + C$

Q6 (iii) $\int \frac{x^2 - 2x + 6}{\sqrt{x}}\,dx = \int \left(x^{3/2} - 2x^{1/2} + 6x^{-1/2}\right)dx$

$= \frac{x^{5/2}}{5/2} - \frac{2x^{3/2}}{3/2} + \frac{6x^{1/2}}{1/2} + C$

$= \frac{2}{5}\sqrt{x^5} - \frac{4}{3}\sqrt{x^3} + 12\sqrt{x} + C$

Q8 $f'(x)\,dx = 2x - 5$

$f(x) = \int 2x - 5 = \frac{2x^2}{2} - 5x + C$

$= x^2 - 5x + C$

$(1, 7)$

$7 = (1)^2 - 5(1) + C$

$7 = 1 - 5 + C$

$7 = -4 + C$

$11 = C$

$\Rightarrow f(x) = x^2 - 5x + 11$

Q9 $\int (6x+5)\,dx = 19$ $x = 2$.

$\frac{6x^2}{2} + 5x + c = 19$

$3x^2 + 5x + c = 19$

at $x=2$: $3(2)^2 + 5(2) + c = 19$

$12 + 10 + c = 19$

$c = 19 - 22$

$c = -3$

Q11 (ii) $\frac{dy}{dx} = 3 - x^2$

$y = 3x - \frac{x^3}{3} + c$ $(y = 2,\ x = 3)$

$2 = 3(3) - \frac{(3)^3}{3} + c$

$2 = 9 - 9 + c$

$2 = c$

$\Rightarrow y = 3x - \frac{x^3}{3} + 2.$

Q12 $\frac{dV}{dt} = t^2 - t$

$\Rightarrow V = \int t^2 - t = \frac{t^3}{3} - \frac{t^2}{2} + c$

$V = \frac{1}{3}t^3 - \frac{1}{2}t^2 + c$ $[V = 9,\ t = 3]$

$9 = \frac{1}{3}(3)^3 - \frac{1}{2}(3)^2 + c$

$9 = 9 - 4.5 + c$

$4.5 = c$

$\Rightarrow V = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 4.5$

(ii) V when $t = 10$

$V = \frac{1}{3}(10)^3 - \frac{1}{2}(10)^2 + 4.5$

$= \frac{1000}{3} - \frac{100}{2} + \frac{9}{2} = 287.83$

Q13 (i) $f'(x) = 4x + k$ turn pt at $(-2,-1)$

$\Rightarrow \frac{dy}{dx} = 0$ at $x = -2$

$4(-2) + k = 0$

$k = 8$

(ii) $f'(x) = 4x + 8$

$f(x) = \int 4x + 8 = \frac{4x^2}{2} + 8x + C = 2x^2 + 8x + C$

pt $(-2,-1)$

$-1 = \frac{4(-2)^2}{2} + 8(-2) + C$

$-1 = 8 - 16 + C$

$7 = C$

$\Rightarrow f(x) = 2x^2 + 8x - 7$

$\therefore$ Cuts y axis at $y = 7$ or $(0,7)$

Q14 $\frac{dy}{dx} = 2x + k$ Tangent at $(3,6)$

$(3,6)\ (0,0) \Rightarrow$ Slope $= \frac{-6}{-3} = 2$.

(i) Slope $\left(\frac{dy}{dx}\right) = 2$ at $x = 3$.

$\therefore\ 2 = 2(3) + k$

$\Rightarrow\ -4 = k$

(ii) $\frac{dy}{dx} = 2x - 4$

$\Rightarrow y = \int 2x - 4 = \frac{\cancel{2}x^2}{\cancel{2}} - 4x + C$

contains pt $(3,6)$

$\Rightarrow 6 = (3)^2 - 4(3) + C$

$6 = 9 - 12 + C$

$9 = C$

$\therefore\ y = x^2 - 4x + 9$