Ex 4.4

(2) $\int_1^3 (3x^2 - 2x)\,dx = \left[\frac{3x^3}{3} - \frac{2x^2}{2}\right]_1^3$

$= \left((3)^3 - (3)^2\right) - \left[(1)^3 - (1)^2\right]$

$= 18 - 0 = 18$

(5) $\int_1^3 (x^2 - x + 1)\,dx = \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_1^3$

$\left(\frac{(3)^3}{3} - \frac{(3)^2}{2} + 3\right) - \left(\frac{(1)^3}{3} - \frac{(1)^2}{2} + 1\right) =$

$7\frac{1}{2} - \frac{5}{6} = 6\frac{2}{3}$

(8) $\int_1^9 \sqrt{x}\,dx = \int_1^9 x^{\frac{1}{2}}\,dx = \left[\frac{x^{3/2}}{3/2}\right]_1^9 = \left[\frac{2x^{3/2}}{3}\right]_1^9$

$= \left(\frac{2(9)^{3/2}}{3}\right) - \left(\frac{2(1)^{3/2}}{3}\right) = 18 - \frac{2}{3} = \frac{52}{3} = 17\frac{1}{3}$

(11) $\int_0^2 \left(\frac{x^3 - 2x^2 + 4x}{x}\right) dx = \int_0^2 (x^2 - 2x + 4)\,dx$

$= \left[\frac{x^3}{3} - \frac{2x^2}{2} + 4x\right]_0^2$

$= \left(\frac{(2)^3}{3} - (2)^2 + 4(2)\right) - \left(\frac{(0)^3}{3} - (0)^2 + 4(0)\right) = \frac{20}{3} = 6\frac{2}{3}$

Q14 $\int_1^{16} \left(\frac{\sqrt{x} - 4}{\sqrt{x}}\right) dx = \int_1^{16} \left(\frac{x^{\frac{1}{2}} - 4}{x^{\frac{1}{2}}}\right) dx$

$= \int_1^{16} \left(1 - 4x^{-\frac{1}{2}}\right) dx = \left[x - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^{16}$

$= \left[x - 8\sqrt{x}\right]_1^{16}$

$= \left(16 - 8\sqrt{16}\right) - \left(1 - 8\sqrt{1}\right) = -16 + 7 = -9$

Q16 $\int_1^2 (x-1)(x-2)\, dx = -\frac{1}{6}$

$\int_1^2 \left(x^2 - 3x + 2\right) dx = \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_1^2$

$= \left(\frac{(2)^3}{3} - \frac{3(2)^2}{2} + 2(2)\right) - \left(\frac{1^3}{3} - \frac{3(1)^2}{2} + 2(1)\right)$

$\left(\frac{8}{3} - 6 + 4\right) - \left(\frac{1}{3} - \frac{3}{2} + 2\right)$

$\frac{2}{3} - \frac{5}{6} = -\frac{1}{6}$ True.

Q17 $\frac{x^2-16}{2x+8} = \frac{(x+4)(x-4)}{2(x+4)} = \frac{x-4}{2}$

$\int_0^1 \frac{x^2-16}{2x+8}\,dx = \int_0^1 \frac{x-4}{2}\,dx = \int_0^1 \left(\frac{x}{2}-2\right)dx$

$= \left[\frac{x^2}{2(2)} - 2x\right]_0^1 = \left(\frac{(1)^2}{4} - 2(1)\right) - \left(\frac{0^2}{4} - 2(0)\right)$

$= \frac{1}{4} - 2 = -\frac{7}{4} = -1\tfrac{3}{4}$

Q18 $\int_0^k (2x-4)\,dx = -3$ find k.

$\left[\frac{\cancel{2}x^2}{\cancel{2}} - 4x\right]_0^k = -3$

$(k^2-4k) - (0^2-4(0)) = -3$

$k^2 - 4x + 3 = 0$

$(k-3)(k-1) = 0$

$k = 3 \quad k = 1$

Q19 $\int_0^k (x^2-3x)\,dx = 0$

$\left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_0^k = 0$

$\left(\frac{k^3}{3} - \frac{3k^2}{2}\right) - \cancel{\left(\frac{0^3}{3} - \frac{30^2}{2}\right)} = 0$

$2k^3 - 9k^2 = 0$

$k^2(2k-9) = 0$

$k^2 = 0 \qquad 2k = 9$

$k = 0 \qquad k = 9/2$

Since $k > 0$
Sol $k = 9/2$

(20) $x^3 - 8 = (x-2)(x^2+2x+4)$

$$\int_0^2 \left(\frac{x^3-8}{x-2}\right) dx = \int_0^2 (x^2+2x+4)\, dx$$

$$= \left[\frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} + 4x\right]_0^2$$

$$= \left[\frac{(2)^3}{3} + (2)^2 + 4(2)\right] - \cancel{\left(\frac{0^3}{3} + 0^2 + 4(0)\right)}$$

$$= \frac{8}{3} + 4 + 8 \quad = \frac{44}{3} = 14\tfrac{2}{3}$$

(Q21) $\int_0^1 nx^2\, dx = 1 \qquad n\int_0^1 x^2\, dx$

$$= n\left[x^3/3\right]_0^1 = n\left(1^3/3 - 0^3/3\right)$$

$$= n\frac{1}{3} = 1 \Rightarrow n = 3.$$

(Q22) (i) $\int_0^{\pi/4} \cos 2x\, dx = \left[\frac{\sin 2x}{2}\right]_0^{\pi/4}$

$$= \left(\frac{\sin 2(\pi/4)}{2} - \frac{\sin 2(0)}{2}\right) = 1/2.$$

(ii) $\int_0^{\pi/6} (\sin 3x)\, dx = \left[\frac{-\cos 3x}{3}\right]_0^{\pi/6}$

$$= \left(\frac{-\cos 3(\pi/6)}{3}\right) - \left(\frac{-\cos 3(0)}{3}\right) = 0 + \frac{1}{3} = 1/3$$

(iii) $\int_{\pi/3}^{\pi/2} 5 \sin x \, dx = 5 \int_{\pi/3}^{\pi/2} \sin x \, dx$

$= 5 \left[-\cos x \right]_{\frac{\pi}{3}}^{\pi/2} = 5 \left[(-\cos \pi/2) - (-\cos \pi/3) \right]$

$= 5 \left[0 - \left(-\frac{1}{2}\right) \right] = 5\left(\frac{1}{2}\right) = 5/2$

(iv) $\int_0^{\pi/2} (2 \cos x + 1) \, dx$

$= \left[2 . \sin x + x \right]_0^{\pi/2}$

$= \left(2 \sin \pi/2 + \pi/2\right) - \left\{ 2 \sin(0) + 0 \right\}$

$\left(2 (1) + \frac{\pi}{2}\right) - \left\{ 2 (0) + 0 \right\}$

$2 + \pi/2 = 3.57$

Q 23

(i) $\int_0^2 e^{4x} dx = \left[\frac{e^{4x}}{4} \right]_0^2$

$= \left(\frac{e^{4(2)}}{4} \right) - \left(\frac{e^{4(0)}}{4} \right) = \frac{e^8}{4} - \frac{1}{4}$

$= 744.99$

(ii) $\int_{-1}^{1} e^{x+3} dx = \left[e^{x+3} \right]_{-1}^{1}$

$= \left(e^{1+3}\right) - \left(e^{-1+3}\right) = e^4 - e^2$

(iii) $\int_0^1 e^{\frac{x}{2}}\,dx = \left[\frac{e^{x/2}}{\frac{1}{2}}\right]_0^1 = \left[2e^{x/2}\right]_0^1$

$= 2e^{\frac{1}{2}} - 2e^0 = 2e^{\frac{1}{2}} - 2$

(iv) $\int_0^1 \left(e^{-2x}+1\right)dx = \left[\frac{e^{-2x}}{-2} + x\right]_0^1$

$= \left[\frac{e^{-2}}{-2} + (1)\right] - \left(\frac{e^{-2(0)}}{-2} - 0\right)$

$= \left(-\frac{1}{2e^2} + 1\right) - \left(\frac{1}{-2}\right)$

$= -\frac{1}{2e^2} + 1 + \frac{1}{2} = -\frac{1}{2e^2} + \frac{3}{2}$

$= \frac{1}{2}\left[3 - \frac{1}{e^2}\right]$

Q24

(i) $\int_0^1 \left(2e^{x/3}+2\right)dx = \int_0^1 2\left(e^{x/3}+1\right)dx$

$= 2\int_0^1 \left(e^{x/3}+1\right)dx = 2\left[\frac{e^{x/3}}{1/3} + x\right]_0^1$

$= 2\left[3e^{x/3}+x\right]_0^1$

$= 2\left[\left(3e^{1/3}+1\right) - \left(3e^{0/3}+0\right)\right]$

$= 2\left(3e^{1/3}+1-3\right)$

$= 2\left(3e^{1/3}-2\right)$

$= 6e^{1/3}-4$

(ii) $\int_{-2}^{2} \frac{e^x+e^{-x}}{2}\,dx = \frac{1}{2}\int_{-2}^{2} e^x + e^{-x}$

$= \frac{1}{2}\left[e^x + \frac{e^{-x}}{-1}\right]_{-2}^{2} = \frac{1}{2}\left[e^x - e^{-x}\right]_{-2}^{2}$

$= \frac{1}{2}\left[\left(e^2-e^{-2}\right) - \left(e^{-2}-e^2\right)\right]$

$= \frac{1}{2}\left[e^2 - \frac{1}{e^2} - \frac{1}{e^2} + e^2\right]$

$= \frac{1}{2}\left(2e^2 - 2\frac{1}{e^2}\right) = e^2 - \frac{1}{e^2}$

(iii) $\int_1^3 5^x \, dx = \left[\frac{5^x}{\ln 5}\right]_1^3$

$= \left(\frac{5^3}{\ln 5}\right) - \left(\frac{5^1}{\ln 5}\right) = \frac{125}{\ln 5} - \frac{5}{\ln 5} = \frac{120}{\ln 5}$

$= 74.56$

(iv) $\int_0^e 7^x \, dx = \left[\frac{7^x}{\ln 7}\right]_0^e$

$= \frac{7^e}{\ln 7} - \frac{7^0}{\ln 7} = \frac{7^e}{\ln 7} - \frac{1}{\ln 7} = \frac{7^e - 1}{\ln 7}$

Q25 $f(x) = \frac{\cos x}{\sin x}$ Quotient Rule $\left(\frac{-1}{\sin^2 x}\right)$.

$f'(x) = \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x}$

$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$= \frac{-1(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$= \frac{-1(1)}{\sin^2 x} = \frac{-1}{\sin^2 x}$ True

$$\int_{\pi/4}^{\pi/2} \frac{1}{\sin^3 x}\,dx$$

$$= -\left[\frac{\cos x}{\sin x}\right]_{\pi/4}^{\pi/2}$$

$$= -\left[\frac{\cos \pi/2}{\sin \pi/2} - \frac{\cos \pi/4}{\sin \pi/4}\right] = -\frac{0}{1} + \frac{1/\sqrt{2}}{1/\sqrt{2}}$$

$$= 0 + 1 = 1$$

Q26 $\frac{d}{dx} x \sin 3x$ (product rule)

$$= x\left[\cos 3x(3)\right] + \sin 3x(1)$$

$$= 3x \cos 3x + \sin 3x$$

Hence $\int_0^{\pi/6} 3x \cos 3x \, dx$

$$\int (3x \cos 3x + \sin 3x)\,dx = x \sin 3x$$

$$\int_0^{\pi/6} (3x \cos 3x)\,dx + \int_0^{\pi/6} (\sin 3x)\,dx = \left[x \sin 3x\right]_0^{\pi/6}$$

$$\int_0^{\pi/6} (3x \cos 3x)\,dx = \left[x \sin 3x\right]_0^{\pi/6} - \int_0^{\pi/6} (\sin 3x)\,dx$$

$$\int_0^{\pi/6} (3x \cos 3x)\,dx = \left[\left(\pi/6 \sin 3\pi/6\right) - \left(0 \sin 3(0)\right)\right] - \left[\left(-\frac{\cos 3(\frac{\pi}{6})}{3}\right) - \frac{-\cos 3(0)}{3}\right]$$

$$\int_0^{\pi/6} (3x \cos 3x)\,dx = \left(\pi/6 \sin \pi/2\right) - \left(\frac{0}{3} + \frac{1}{3}\right)$$

$$= \frac{\pi}{6}(1) + \frac{0}{3} - \frac{1}{3} = \frac{\pi}{6} - \frac{1}{3}$$