Ex 4.5

Q1 $A = \int_1^3 (x+2)\,dx = \left[\frac{x^2}{2} + 2x\right]_1^3$

$= \left(\frac{3^2}{2} + 2(3)\right) - \left(\frac{1^2}{2} + 2(1)\right)$

$= \left(\frac{9}{2} + 6\right) - \left(\frac{1}{2} + 2\right)$

$\frac{21}{2} - \frac{5}{2} = 8$ sq units

Q2

$A = \int_0^2 (x^2+1)\,dx = \left[\frac{x^3}{3} + x\right]_0^2$

$= \left(\frac{2^3}{3} + 2\right) - \cancel{\left(\frac{0^3}{3} + 0\right)}$

$\frac{8}{3} + 3 = \frac{14}{3} = 3\frac{2}{3}$ sq units

Q3

$A = \int_0^3 (x^2 - 3x)\,dx = \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_0^3$

$= \left(\frac{3^3}{3} - \frac{3(3)^2}{2}\right) - \cancel{\left(\frac{0^3}{3} - \frac{3(0)^2}{2}\right)}$

$9 - \frac{27}{2} = -\frac{9}{2} \Rightarrow$ Area $= 4\frac{1}{2}$ Sq units

(Take absolute value)

Q5 $\quad \text{Area} = \int_0^2 (x^3 - 6x^2 + 8x)\,dx + \int_2^4 (x^3 - 6x^2 + 8x)\,dx$

$$\left[\frac{x^4}{4} - \frac{\cancel{6}^2 x^3}{\cancel{3}} + \frac{\cancel{8}^4 x^2}{\cancel{2}}\right]_0^2 + \left[\frac{x^4}{4} - \frac{\cancel{6}^2 x^3}{\cancel{3}} + \frac{\cancel{8}^4 x^2}{\cancel{2}}\right]_2^4$$

$$\left[\left(\frac{2^4}{4} - 2(2)^3 + 4(2)^2\right) - 0\right] + \left[\left(\frac{4^4}{4} - 2(4)^3 + 4(4)^2\right) - \left(\frac{2^4}{4} - 2(2)^3 + 4(2)^2\right)\right]$$

$$4 \quad + \quad [0 - 4]$$

$$4 \quad + \quad 4 \quad = \quad 8 \text{ sq units}$$

Q7 $\quad \text{Area} = \int_{-1}^0 (3x - x^2)\,dx + \int_0^3 (3x - x^2)\,dx + \int_3^4 (3x - x^2)\,dx$

$$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_{-1}^0 + \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 + \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_3^4$$

$$\left[\left(\frac{3(0)^2}{2} - \frac{(0)^3}{3}\right) - \left(\frac{3(-1)^2}{2} - \frac{(-1)^3}{3}\right)\right] + \left[\left(\frac{3(3)^2}{2} - \frac{(3)^3}{3}\right) - \left(\frac{3(0)^2}{2} - \frac{0^3}{3}\right)\right]$$

$$+ \left[\left(\frac{3(4)^2}{2} - \frac{4^3}{3}\right) - \left(\frac{3(3)^2}{2} - \frac{3^3}{3}\right)\right]$$

$$\left(0 - \frac{11}{6}\right) + \left(\frac{9}{2} - 0\right) + \left(\frac{8}{3} - \frac{9}{2}\right)$$

$$\frac{-11}{6} + \frac{9}{2} - \frac{11}{6} \longrightarrow \text{Take absolute Value}$$

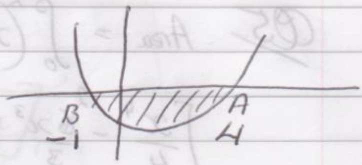
$$= \frac{11}{6} + \frac{9}{2} + \frac{11}{6}$$

$$= \frac{49}{6} = 8\tfrac{1}{6} \text{ sq units}$$

Q9 $y = x^2 - 3x - 4$

$(x-4)(x+1) = 0$

$x = 4 \quad x = -1$



$$\text{Area} = \int_{-1}^{4} (x^2 - 3x - 4)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{3x^2}{2} - 4x\right]_{-1}^{4}$$

$$= \left(\frac{(4)^3}{3} - \frac{3(4)^2}{2} - 4(4)\right) - \left(\frac{(-1)^3}{3} - \frac{3(-1)^2}{2} - 4(-1)\right)$$

$$\left(\frac{64}{3} - 24 - 16\right) - \left(\frac{-1}{3} - \frac{3}{2} + 4\right)$$

$$\frac{-56}{3} - \frac{17}{6} = -\frac{125}{6}$$

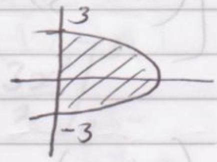
absolute value $\Rightarrow$ Area $= 20\frac{5}{6}$ sq units

Q10

$x = 9 - y^2$

$(3+y)(3-y)$

$y = -3 \quad y = 3$



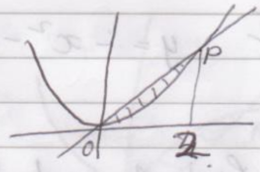
$$\text{Area} = \int_{-3}^{3} (9 - y^2)\,dy$$

$$= \left[9y - \frac{y^3}{3}\right]_{-3}^{3}$$

$$= \left(9(3) - \frac{(3)^3}{3}\right) - \left(9(-3) - \frac{(-3)^3}{3}\right)$$

$$18 - (-18) = 18 + 18 = 36 \text{ sq units}$$

Q13 $y = x^2$ $y = 2x$



find P: $x^2 = 2x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \quad x = 2$

P: $y = 2x \Rightarrow y = 2(2) \Rightarrow y = 4$

$P(2, 4)$

Area $= \int_0^4 2x\,dx - \int_0^4 x^2\,dx$

$= \left[\frac{\cancel{2}x^2}{\cancel{2}}\right]_0^2 - \left[\frac{x^3}{3}\right]_0^2$

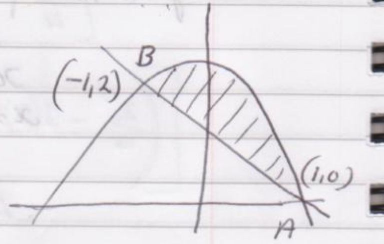
$= (2^2 - 0^2) - \left(\frac{2^3}{3} - \frac{0^3}{3}\right)$

$= 4 - \frac{8}{3} = \frac{4}{3}$ sq units

Q15 $\quad y = -x^2 - x + 2 \qquad x + y - 1 = 0$

$y = 1 - x$

(i) find A and B



$$-x^2 - \cancel{x} + 2 = 1 - \cancel{x}$$
$$-x^2 + 1 = 0$$
$$x^2 - 1 = 0$$
$$(x+1)(x-1) = 0$$
$$x = -1 \quad x = 1$$

$y = 1 - x \Rightarrow y = 2 \quad y = 0$

B $(-1, 2)$ $\qquad$ A $(1, 0)$

(ii) $\text{Area} = \int_{-1}^{1} (-x^2 - x + 2)\,dx - \int_{-1}^{1} (1 - x)\,dx$

$$= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 2x\right]_{-1}^{1} - \left[x - \frac{x^2}{2}\right]_{-1}^{1}$$

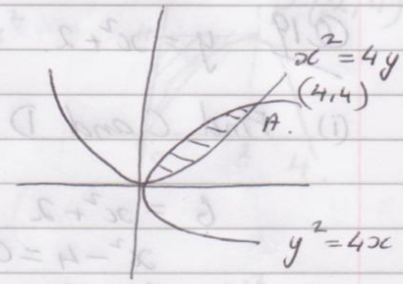
$$\left[\left(-\frac{(1)^3}{3} - \frac{(1)^2}{2} + 2(1)\right) - \left(-\frac{(-1)^3}{3} - \frac{(-1)^2}{2} + 2(-1)\right)\right] - \left[\left(1 - \frac{1^2}{2}\right) - \left(-1 - \frac{(-1)^2}{2}\right)\right]$$

$$\left[\left(\frac{7}{6}\right) - \left(-\frac{13}{6}\right)\right] - \left[\left(\frac{1}{2}\right) - \left(-\frac{3}{2}\right)\right]$$

$$\left(\frac{7}{6} + \frac{13}{6}\right) - \left(\frac{1}{2} + \frac{3}{2}\right)$$

$$\frac{10}{3} - 2 = \frac{4}{3} \text{ sq units.}$$

Q17 $y^2 = 4x$ $x^2 = 4y$



(i) Find A.

$y^2 = 4x$ $x^2 = 4y$

$y = \sqrt{4x}$ $y = \frac{x^2}{4}$

$\sqrt{4x} = \frac{x^2}{4}$ (sq both sides)

$4x = \frac{x^4}{16}$

$64x = x^4$

$x^4 - 64x = 0$

$x(x^3 - 64) = 0$

$x(x-4)(x^2+4x+16) = 0$

$x = 0$ $x = 4$

At point A $x = 4$. $y = \frac{x^2}{4} \Rightarrow y = 4 \Rightarrow A = (4,4)$

(ii) Area $= \int_0^4 (\sqrt{4x})\,dx - \int_0^4 \left(\frac{x^2}{4}\right) dx$

$= \int_0^4 (2x^{\frac{1}{2}})\,dx - \frac{1}{4}\int_0^4 x^2\,dx$

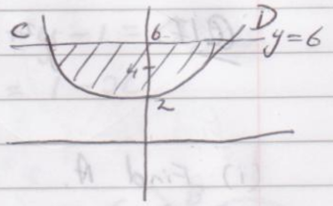
$= \left[2\frac{x^{3/2}}{3/2}\right]_0^4 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4 = \left[\frac{4x^{3/2}}{3}\right]_0^4 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$

$= \left[\left(\frac{4(4)^{3/2}}{3}\right) - \left(\frac{4(0)^{3/2}}{3}\right)\right] - \frac{1}{4}\left[\left(\frac{4^3}{3}\right) - \left(\frac{0^3}{3}\right)\right]$

$\left(\frac{32}{3} - 0\right) - \frac{1}{4}\left(\frac{64}{3}\right)$

$\frac{32}{3} - \frac{16}{3} = \frac{16}{3} = 5\frac{1}{3}$ sq units.

Q19 $y = x^2 + 2$. $y = 6$



(i) Find C and D

$$6 = x^2 + 2$$
$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$
$$x = -2 \quad x = 2.$$

$C(-2,6)$ $D(2,6)$

(ii) $\text{Area} = \int_{-2}^{2} 6\,dx - \int_{-2}^{2} (x^2+2)\,dx$

$$= \left[6x\right]_{-2}^{2} - \left[\frac{x^3}{3} + 2x\right]_{-2}^{2}$$

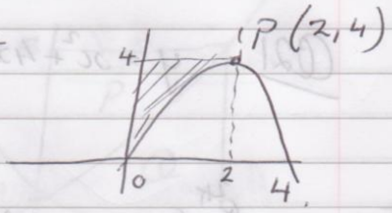
$$\left[6(2) - 6(-2)\right] - \left[\left(\frac{2^3}{3} + 2(2)\right) - \left(\frac{(-2)^3}{3} + 2(-2)\right)\right]$$

$$\left[12 + 12\right] - \left[\frac{20}{3} - \left(-\frac{20}{3}\right)\right]$$

$$24 - \left[\frac{40}{3}\right]$$

$$= \frac{32}{3} = 10\tfrac{2}{3} \text{ sq units.}$$

Q20 $\quad y = x(4 - x) = 4x - x^2$



(i) find P.

P is Max pt.

$$\Rightarrow \frac{dy}{dx} = 4 - 2x = 0$$

$$2x = 4$$

$$x = 2. \qquad y = 4(2) - (2)^2 = 4$$

$$\Rightarrow P(2, 4)$$

(ii) Shaded area = area Rec$(y=4 \; x=2)$ $-$ $\left[\text{Area Under Curve}\right]_0^2$

$$= (2 \times 4) - \int_0^2 (4x - x^2)\,dx$$

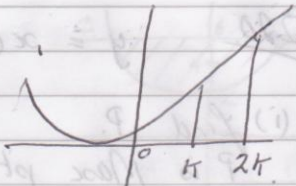
$$= 8 - \left[\frac{\cancel{4}^{2} x^2}{\cancel{2}} - \frac{x^3}{3}\right]_0^2$$

$$= 8 - \left[\left(2(2)^2 - \frac{(2)^3}{3}\right) - \left(2(0)^2 - \frac{0^3}{3}\right)\right]$$

$$= 8 - \left(8 - 8/3\right)$$

$$= 8 - \frac{16}{3} = \frac{8}{3} = 2\tfrac{2}{3} \text{ sq units.}$$

Q21 $y = x^2 + 4x + 4$



$$\int_0^{2k} (x^2 + 4x + 4)\,dx = 4\int_0^{k} (x^2 + 4x + 4)\,dx$$

$$\left[\frac{x^3}{3} + \frac{\cancel{4}x^2}{\cancel{2}} + 4x\right]_0^{2k} = 4\left[\frac{x^3}{3} + 2x^2 + 4x\right]_0^{k}$$

$$\left[\left(\frac{(2k)^3}{3} + 2(2k)^2 + 4(2k)\right) - 0\right] = 4\left[\left(\frac{k^3}{3} + 2k^2 + 4k\right) - 0\right].$$

$$\frac{8k^3}{3} + 8k^2 + 8k = 4\left(\frac{k^3}{3} + 2k^2 + 4k\right)$$

$$\frac{8k^3}{3} + \cancel{8k^2} + 8k = \frac{4k^3}{3} + \cancel{8k^2} + 16k \quad (\times 3)$$

$$8k^3 + 24k = 4k^3 + 48k$$

$$4k^3 - 24k = 0 \quad (\div 4)$$

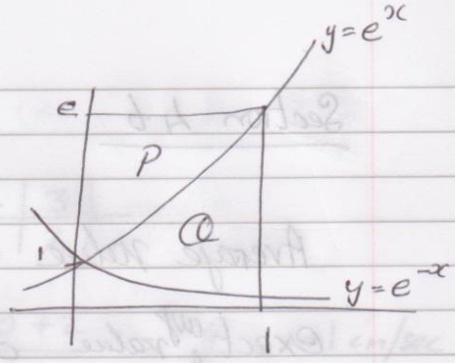
$$k^3 - 6k = 0$$

$$k(k^2 - 6) = 0$$

$k = 0$ $\qquad k^2 = 6$

$$k = \sqrt{6}.$$

Q24 $\quad y = e^{x} \quad y = e^{-x}$



(i) Area $P = \int_0^1 e\,dx - \int_0^1 e^{x}\,dx$

$= [ex]_0^1 - [e^{x}]_0^1$

$= [e(1) - e(0)] - [e^{1} - e^{0}]$

$= (e) - (e-1)$

$= (e - e + 1)$ sq units

(ii) Area $Q = \int_0^1 e^{x}\,dx - \int_0^1 e^{-x}\,dx$

$= [e^{x}]_0^1 - \left[\frac{e^{-x}}{-1}\right]_0^1$

$= [e^{x}]_0^1 - [-e^{-x}]_0^1$

$= (e^{1} - e^{0}) - (-e^{-1} - (-e^{0}))$

$= (e - 1) - \left(\frac{-1}{e} + 1\right)$

$= e - 1 + \frac{1}{e} - 1$

$= \left(e + \frac{1}{e} - 2\right)$ sq units