# Revision Exercises

**Q4** roots: $x = -7$ and $x = 0$

$\Rightarrow$ Eqn: $y = a(x+7)x$.

$(4,4)$ $\quad 4 = a(4+7)(4)$

$4 = a(44)$

$\frac{4}{44} = a$

$\frac{1}{11} = a$

$\Rightarrow$ Eqn is: $\quad y = \frac{1}{11}(x+7)(x)$

$y = \frac{x}{11}(x+7)$

**Q5**

$g(x) = 5 + \frac{x}{2}$

$g^{-1}(x)$ $\qquad y = 5 + \frac{x}{2}$

$y - 5 = \frac{x}{2}$

$2(y-5) = x$

$g^{-1}(x) = 2(x-5)$

(i) $g^{-1}(-2) = 2((-2)-5) = -14$

(ii) $g(x) = g^{-1}(x)$

$5 + x/2 = 2(x-5)$

$5 + x/2 = 2x - 10$

$5 + 10 = 2x - x/2$

$15 = \frac{3}{2}x$

$15 \times \frac{2}{3} = x$ $\qquad x = 10$

Q8 $\quad y = 2m^x$

(i) $(3, 54)$

$$54 = 2m^3$$
$$27 = m^3$$
$$\sqrt[3]{27} = m$$
$$3 = m.$$

(ii) $P(0, y)$

$$y = 2.3^x$$
$$y = 2 \cdot 3^0$$
$$y = 2(1) = 2.$$

$P(0,2)$

Q9 (i) $\displaystyle \lim_{x \to 0} \frac{5x-4}{3+x} = \frac{5(0)-4}{3+(0)} = \frac{-4}{3}$

(ii) $\displaystyle \lim_{x \to 1} \frac{x^2-3x+2}{x-1} = \frac{\cancel{(x-1)}(x-2)}{\cancel{x-1}} = \lim_{x \to 1} x-2 = -1$

(iii) $\displaystyle \lim_{x \to 4} \frac{x^3-64}{x^2-16} = \lim_{x \to 4} \frac{\cancel{(x-4)}(x^2+4x+16)}{(x+4)\cancel{(x-4)}} = \lim_{x \to 4} \frac{x^2+4x+16}{x+4}$

$$= \frac{(4)^2+4(4)+16}{(4)+4} = \frac{48}{8} = 6.$$

Q12 Graph A $\Rightarrow y = \left(\frac{1}{2}\right)^x$

Graph B $\Rightarrow y = \left(1/3\right)^x$

Graph C $\Rightarrow y = 5^x$

Graph D $\Rightarrow y = 2^x$

Q13 (i) $x = 2$

(ii) $x = 3$

(iii) $x < 4$.

Q15 $f(x) = x^2 + 3$ $\quad g(x) = x + 4$.

(i) $fg(x)$

$g(x) = x+4$

$f(g(x)) = f(x+4) = (x+4)^2 + 3$

$= x^2 + 8x + 16 + 3$

$= x^2 + 8x + 19$.

$gf(x)$

$f(x) = x^2 + 3$

$g(f(x)) = g(x^2+3) = x^2 + 3 + 4$

$= x^2 + 7$

(ii) $fg(x) + gf(x) = 0$ has no real roots

$\Rightarrow b^2 - 4ac < 0$

$x^2 + 8x + 19 + x^2 + 7 = 0$

$2x^2 + 8x + 26 = 0$

$x^2 + 4x + 13 = 0$

$b^2 - 4ac < 0$

$(4)^2 - 4(1)(13) < 0$

$16 - 52 < 0$

$-36 < 0$ True.

# Revision Exercise Advanced.

**Q1** $f(x) = x-1 \qquad g(x) = 2x^2 - x - 1 \qquad h(x) = \log_3 x$

(i) $hf(x)$ $\qquad f(x) = x-1$

$$h(f(x)) = h(x-1) = \log_3 (x-1)$$

$hg(x)$ $\qquad g(x) = 2x^2 - x - 1$

$$h(g(x)) = h(2x^2 - x - 1) = \log_3 (2x^2 - x - 1)$$

(ii) $hg(x) - hf(x) = 2$

$$\log_3 (2x^2 - x - 1) - \log_3 (x-1) = 2$$

$$\log_3 \frac{2x^2 - x - 1}{x-1} = 2$$

$$\frac{2x^2 - x - 1}{x-1} = 3^2$$

$$2x^2 - x - 1 = 9x - 9$$
$$2x^2 - 10x + 8 = 0$$
$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0$$
$$x = 4 \qquad x = 1$$

Q5 $V = L \times w \times h$.

$L = (24-2x) \qquad w = (18-2x) \qquad h = x$

$V = (24-2x)(18-2x)(x)$

* for Vol to exist $(18-2x) > 0$

$-2x > -18$

$2x < 18$

$x < 9$

$\Rightarrow$ Domain $0 < x < 9$.

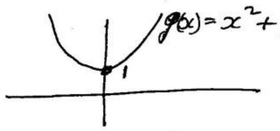
Q6 (ii) (a) $x = 1$ (b) Domain $= R \setminus \{1\}$

(c) Range $= R \setminus \{0\}$ (d) Yes - horizontal test.

Q8

$f: x \Rightarrow 3x - 1 \qquad g: x \Rightarrow x^2 + 1$

(a) $g: x \Rightarrow x^2 + 1$




$\Rightarrow$ Range is $y \geq 1$ or $[1, \infty)$

(b) $gf(x) \Rightarrow g(3x-1) = (3x-1)^2 + 1 = 9x^2 - 6x + 1 + 1 = 9x^2 - 6x + 2$

$fg(x) \Rightarrow f(x^2+1) = 3(x^2+1) - 1 = 3x^2 + 3 - 1 = 3x^2 + 2$

$gf(x) = fg(x)$

$9x^2 - 6x + 2 = 3x^2 + 2$

$6x^2 - 6x = 0$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ and $x = 1$.

(c) $|f(x)| = 8$

$|3x-1| = 8$ sq both sides

$9x^2 - 6x + 1 = 64$

$9x^2 - 6x - 63 = 0$

$3x^2 - 2x - 21 = 0$

$(3x + 7)(x - 3) = 0$

$x = -7/3 \qquad x = 3$

(d) $h: x \Rightarrow x^2 + 3x$ one to one $\Rightarrow$ Injective

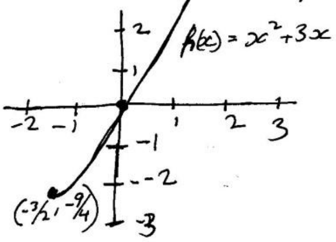
$\Rightarrow$ from Min Point.

find Min pt by complete the square.

$x^2 + 3x + \frac{9}{4} - \frac{9}{4}$

$(x + 3/2)^2 - 9/4$

$\Rightarrow$ min point is $(-3/2, -9/4)$ $\Rightarrow q = -3/2$



Q9 (i) $y = k(x-1)^2(x+t)$ roots at $x=1$ $x=5$.
$\Rightarrow t=-5$

$y = k(x-1)^2(x-5)$

$(0,10)$

$10 = k(0-1)^2(0-5)$

$10 = k(1)(-5)$

$10 = -5k$

$-2 = k.$

(ii) $\lim_{x\to 3} \frac{x^2+2x-15}{x^2-9} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x+5)}{(x+3)\cancel{(x-3)}}$

$= \lim_{x\to 3} \frac{x+5}{x+3} = \frac{3+5}{3+3} = 8/6 = 4/3.$

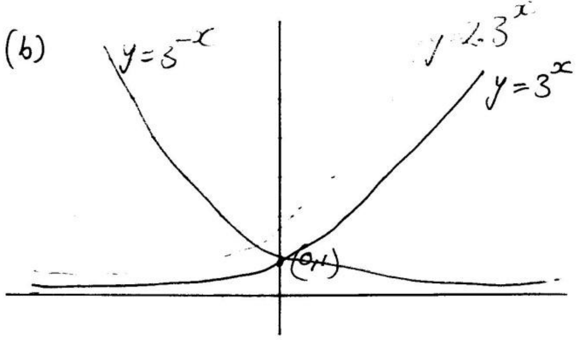
Q10

(i) $f(x) = +\sqrt{25-x^2}$ Domain $-5 \le x \le 5$.

$\Rightarrow$ range is $[0, 5]$

(ii) (a) $y = 3^x$ cuts $y$ at $(0,1)$

(b)



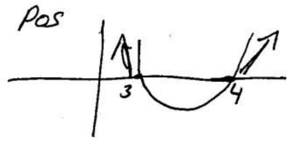
Q12 (i) $f(x) = \sqrt{x^2 - 7x + 12}$

To exist the $\sqrt{\ }$ must be pos

$x^2 - 7x + 12 > 0$

$(x-3)(x-4)$

$x = 3 \quad x = 4$




$\Rightarrow x \leq 3$ and $x \geq 4$

(ii) (a) $y = \sin x$ is a function as vertical test will only intersect graph once.

(b) No (horizontal test)

(c) Yes (horizontal test)

(d) Injective if domain restricted to $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$.

# Revision Exercise — Extended-Response

**Q1** (a) $x^2 + 4x - 2 = (x+a)^2 + b$.

$x^2 + 4x + 4 - 4 - 2$

$(x+2)^2 - 6 \Rightarrow a = 2$ and $b = -6$

Turning pt is $(-2, -6)$

(b) $y = x^2 + 4x - 2$

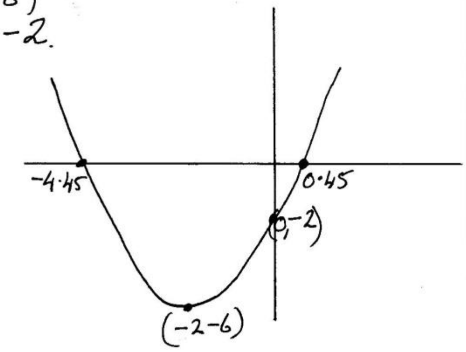
$$x = \frac{-4 \pm \sqrt{16 - 4(1)(-2)}}{2(1)} = \frac{-4 \pm \sqrt{24}}{2}$$

$$= \frac{-4 \pm 2\sqrt{6}}{2} = -2 \pm \sqrt{6} \to 0 \cdot 45,\ -4 \cdot 45$$

$\Rightarrow$ Cuts $x$ axis at $0 \cdot 45$ and $-4 \cdot 45$

min pt is $(-2, -6)$

cuts $y$ at $y = -2$.



(c) $\quad x^2+4x-2.$

$b^2-4ac$

$4^2-4(1)(-2) \quad = \quad 16+8 \quad = 24.$

$b^2-4ac > 0 \quad \Rightarrow$ has 2 real roots.

Hence graph in (b) cuts $x$ axis in 2 places

(d) $\quad x^2+4x+k=0$

No real roots $\quad \Rightarrow \quad b^2-4ac<0$

$4^2-4(1)(k)<0$

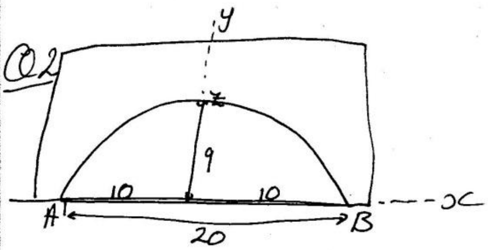
$16-4k<0$

$-4k<-16$

$4k>16 \quad$ (Reverse inequality)

$k>4.$

Q2




$|OA| = |OB|$

arch : $\quad y = ax^2+b.$

arch cuts $x$ axis at $(10,0)$ and $(-10,0)$

cuts $y$ axis at $(0,9)$

$(10,0) \quad 0 = a(10)^2+b$

$0 = 100a+b$

$(0,9) \quad 9 = a(0)^2+b$

$\Rightarrow b=9$

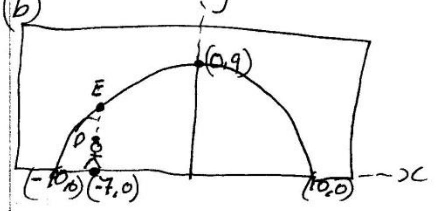
$0 = 100a+9$

$-9 = 100a$

$-0.09 = a$

$\Rightarrow$ arch: $\boxed{y = -0.09x^2+9}$

(b)




Man standing at $(-7,0)$

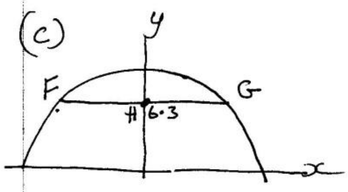
$(-7,0)$

$$y = -0\cdot09x^2 + 9$$
$$y = -0\cdot09(-7)^2 + 9$$
$$y = 4\cdot59.$$

$\Rightarrow$ Point $E$ is $(-7, 4\cdot59)$

$\Rightarrow$ Height from ground is $4\cdot59$.
Man is $1\cdot8$ m height $\Rightarrow |DE| = 4\cdot59 - 1\cdot8 = 2\cdot79$ m

(c)




Height is $6\cdot3$ = y co-ordinate

$$6\cdot3 = -0\cdot09x^2 + 9$$
$$0\cdot09x^2 = 9 - 6\cdot3$$
$$x^2 = \frac{9-6\cdot3}{0\cdot09}$$
$$x^2 = 30$$
$$x = \pm\sqrt{30}$$
$$x = \pm 5\cdot477.$$

$\Rightarrow$ Distance from F to H is $5\cdot477$ and
Distance from H to G is $5\cdot477$

$\Rightarrow |FG| = 10\cdot954$ m

Q3 (a) $y = a\log_2(x-b)$

$(5,2)$ $2 = a\log_2(5-b)$

$\frac{2}{a} = \log_2(5-b)$

$2^{2/a} = 5-b$

$(2^2)^{1/a} = 5-b$

$4^{1/a} = 5-b$

$b = 5-4^{1/a}$

$(7,4)$ $4 = a\log_2(7-b)$

$4/a = \log_2(7-b)$

$2^{4/a} = 7-b$

$(2^2)^{2/a} = 7-b$

$(4)^{2/a} = 7-b.$

$(4^{1/a})^2 = 7-b$

$b = 7-(4^{1/a})^2$

$\Rightarrow \quad 5-4^{1/a} = 7-(4^{1/a})^2$

Let $y = 4^{1/a}$ : $\quad 5-y = 7-y^2$

$y^2-y-2=0$

$(y-2)(y+1)=0$

$y=2 \qquad y=-1$

$2 = 4^{1/a}$

$2^1 = 2^{2/a}$

$1 = 2/a$

$a = 2.$

Sub back in. $\quad 2 = a\log_2(5-b)$

$2 = 2\log_2(5-b)$

$1 = \log_2(5-b)$

$2^1 = 5-b$

$b = 5-2$

$b = 3$

(b) Statement (iii) is untrue.

Q4 (a) $y = f(x)$ $y = -f(x-2)$

Inverse shift 2 to right

$\Rightarrow$ Graph D

(b)(i) $N = N_0 e^{kt}$

(a) $N_0 = 20,000$ (given in Q)

(b) ↓ by 20% $\Rightarrow$ after 1yr $(t=1)$ $N = 20,000 - 4000$
$= 16,000$

$$16,000 = 20000\, e^{k(1)}$$
$$\frac{16000}{20000} = e^{k}$$
$$0.8 = e^{k}$$
$$\ln 0.8 = k \ln e$$
$$\ln 0.8 = k(1)$$
$$-0.2231 = k$$

b (ii) $N = 20,000\, e^{-0.2231t}$

$$5,000 = 20,000\, e^{-0.2231t}$$
$$\frac{5000}{20000} = e^{-0.2231t}$$
$$0.25 = e^{-0.2231t}$$
$$\ln 0.25 = -0.2231t \ln e$$
$$+1.386 = +0.2231t$$
$$\frac{1.386}{0.2231} = t$$
$$6.21 = t$$

6.2 years $= t$.

Q5 $f(x) = x^3$ $g(x) = \frac{1}{x-3}$

(a) Range of $f(x) = R$

(b) (i) $fg(x)$

$$g(x) = \frac{1}{x-3}$$

$$f\left(\frac{1}{x-3}\right) = \left(\frac{1}{x-3}\right)^3 = \frac{1}{(x-3)^3}$$

(ii) $fg(x) = 64$

$$\frac{1}{(x-3)^3} = 64$$

$$\frac{1}{64} = (x-3)^3$$

$$\sqrt[3]{1/64} = x-3$$

$$\frac{1}{4} = x-3$$

$$\frac{1}{4} + 3 = x$$

$$3\tfrac{1}{4} = x$$

(c) (i) $g^{-1}(x)$

$$y = \frac{1}{x-3}$$

$$x - 3 = \frac{1}{y}$$

$$x = \frac{1}{y} + 3.$$

$$\Rightarrow g^{-1}(x) = \frac{1}{x} + 3 = \frac{1+3x}{x}.$$

(ii) range of $g^{-1}(x)$ = domain of $g(x)$

$$= R,\ x \neq 3.$$

(iii) $gg^{-1}(x)$.

$$g^{-1}(x) = \frac{1+3x}{x}$$

$$g\left(\frac{1+3x}{x}\right) = \frac{1}{\frac{1+3x}{x} - 3} = \frac{1}{\frac{1+3x-3x}{x}}$$

$$= \frac{1}{1/x} = 1 \cdot \frac{x}{1} = x.$$

(iv) Not continuous at $x = 0$.

Q6 $M = Ae^{-pt}$

(i) $A = €130,000$

(ii) $122000 = 130000\,e^{-p(1)}$

$\frac{122000}{130000} = e^{-p}$

$\frac{61}{65} = e^{-p}$

$\ln \frac{61}{65} = -p \ln e$

$\ln \frac{61}{65} = -p\,(1)$

$-0.0635 = -p$

$p = 0.0635$

$p \simeq 0.064$ to 2 sig figs.

(iii) end 2011 $\Rightarrow t = 6$ yrs

$M = 130,000\,e^{-0.064(6)}$

$= 130,000\,e^{-0.384}$

$= 88547.085$

$= €88500$ to nearest €100.

~~Q7~~ (a) $y = \log_5(x-2) \Rightarrow$ Graph (B)

~~(b)~~ check with $(7,1)$ $\quad 1 = \log_5(7-2)$

$1 = \log_5 5$ True

check with $(3,0)$ $\quad 0 = \log_5(3-2)$

$0 = \log_5 1$ True.

(b) $y = 4^x \qquad y = 3^{2-x}$

(i) Intersect $\Rightarrow$

$$4^x = 3^{2-x}$$
$$x \log 4 = (2-x)\log 3$$
$$x \log 4 = 2\log 3 - x\log 3$$
$$x\log 4 + x\log 3 = 2\log 3$$
$$x(\log 4 + \log 3) = 2\log 3$$
$$x(\log(4)(3)) = 2\log 3$$
$$x \log 12 = 2\log 3$$
$$x = \frac{2\log 3}{\log 12} = \frac{\log 3^2}{\log 12} = \frac{\log_a 9}{\log_a 12}$$

(ii) $x = \dfrac{\log_a 9}{\log_a 12} = 0.8842$

$y = 4^x$

$y = 4^{0.8842}$

$y = 3.40689$.

$T(0.88, 3.41)$

$\underline{\underline{Q8}}$ $y = x^2 - 4x + 5$

(a) $\underbrace{x^2 - 4x + 4}\, - 4 + 5$

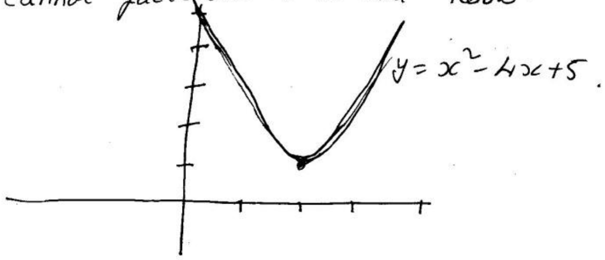
$(x-2)^2 + 1$

Turning pt is $(+2, +1)$

$x^2 - 4x + 5 = 0$ cuts $y$ at 5

$(x \quad)(x \quad) = 0$

cannot factorise $\Rightarrow$ no real Roots.




(b) $y = x^2 - 4x + 5$

$y = (x-2)^2 + 1$

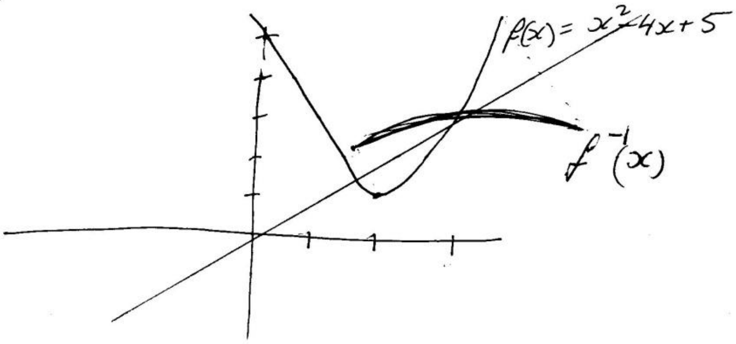
$y - 1 = (x-2)^2$

$\sqrt{y-1} = x - 2$

$\sqrt{y-1} + 2 = x$

$\Rightarrow f^{-1}(x) = \sqrt{x-1} + 2$

(c)



Q9 $\quad C = C_0 e^{-kt}$

(i) $C_0 = 5$

$2.8 = 5e^{-k(1)}$

$\frac{2.8}{5} = e^{-k}$

$0.56 = e^{-k}$

$\ln 0.56 = -k \ln e$

$\ln 0.56 = -k\,(1)$

$-.5798 = -k$

$k = 0.5798$

(ii) $0.2 = 5e^{-0.5798t}$

$\frac{0.2}{5} = e^{-0.5798t}$

$0.04 = e^{-0.5798t}$

$\ln 0.04 = (-0.5798t) \underset{(1)}{\ln e}$.

$\frac{\ln 0.04}{-0.5798} = t$

$5.5517 = t$

$\Rightarrow 5.6$ years.