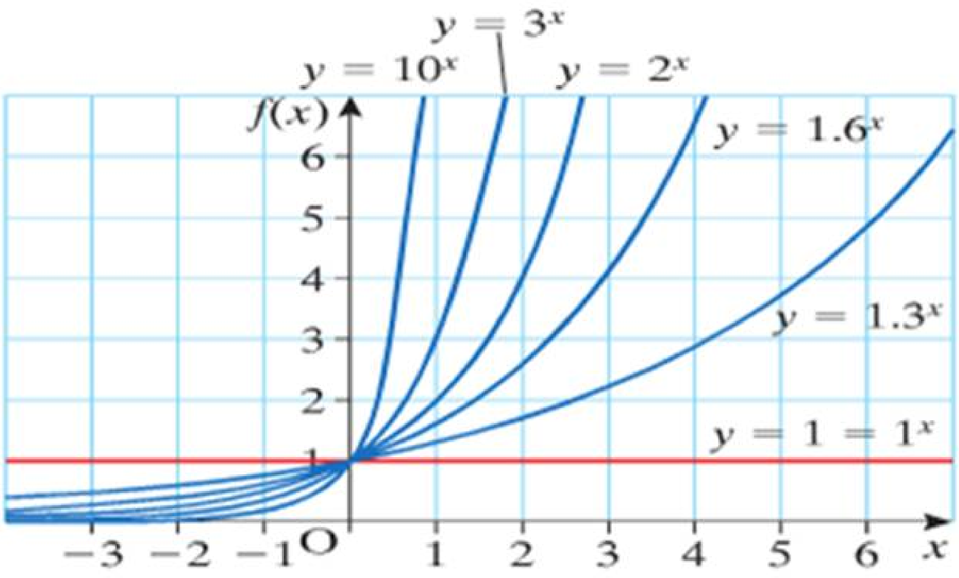
Section 7.8 Exponential Functions

Graphs of exponential functions look like the following.



All exponential graphs when $a > 1$ (The base number is greater than one).

- Pass through the point (0,1) as any base number to the power of zero is one.
- The point (1,a) is on all graphs, (the graph goes through the point (1,base number))
  eg: the graph $y = 3^x$ goes through the point (1,3).
- All graphs tend towards the x-axis, but never reaches it.
- When $a > 1$ (the base number greater than 1) the graph is increasing, also the larger the base number the steeper the curve.
- $0 < a < 1$, the base number is a fraction, the graph is decreasing.
- Functions of the form $f(x) = A.a^x$ cuts the y-axis at A

Example 1

A bacterial colony doubles every hour. If 10 bacteria cells were present at the start of an experiment, (i) complete the following table (ii) draw a graph of the number of bacteria present up to 5 hours.

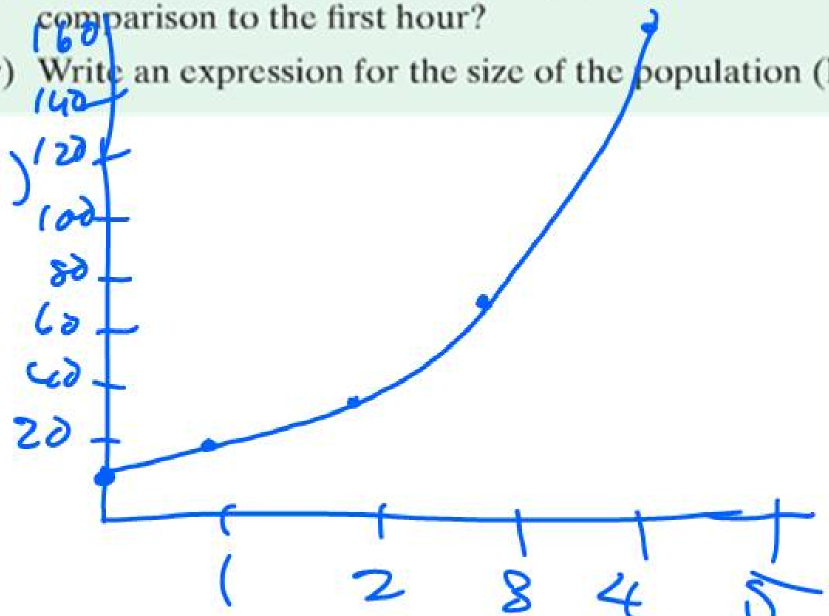
Time in hours	0	1	2	3	4	5
Number of bacteria	10	20	40	80	160	320

(iii) By how many would the population increase in the 6th hour?

(iv) What percentage increase in the population occurred in the 6th hour by comparison to the first hour?

(v) Write an expression for the size of the population (N) after t hours.

(ii)



(iii) 6th hr inc 320inc

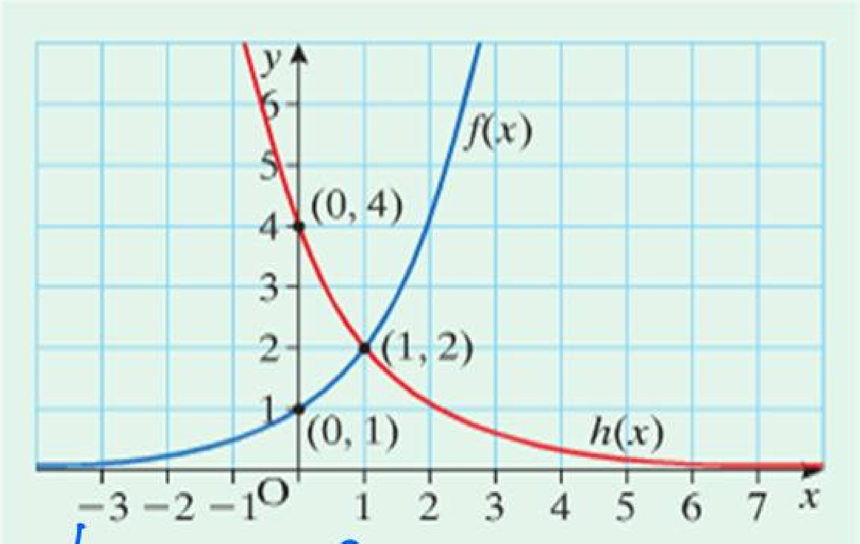
(iv) % Inc from hr 1

$\frac{320}{10} \times 100 = 3200\%$

(v) Write expression.

Example 2

The graphs of two exponential functions, $y = \mathrm{A}a^x$, are given in this diagram. Find the values of A and a for each graph.



$f(x) = A a^x$

from graph $A = 1$

To find a : at $x = 1$ $y = 2$

$1 a^1 = 2$

$a = 2$

$\therefore f(x) = 2^x$

$\underline{h(x)} = A a^x$.

from graph $A = 4$

find a : at $x = 1$ $y = 2$

$4 a^1 = 2$ $a = 2/4$

$a = 1/2 \rightarrow h(x) = 4\left(\frac{1}{2}\right)^x = 4 \cdot 2^{-x}$

Example 3

Given that the intensity of an earthquake is represented by the formula $A = 10^{M}$, and the energy released during a quake by the formula $E \cong 10^{1.5M + 4.8}$, where A is the amplitude and M is the magnitude on the Richter scale, compare
(i) the intensity (ii) the energy of an earthquake of magnitude 6.1 on the Richter scale with a quake of magnitude 4.7.

$$A = 10^{m}$$

$$\frac{A_1 = 10^{6.1} = 1258925.4}{A_1 = 10^{4.7} = 50118.72} = 25 \text{ times greater}$$

A_1 is 25 times greater

Exercise 7.8

Q1

(i) $y = 2^x \Rightarrow$ Graph B

(ii) $y = (0.1)^x \Rightarrow$ Graph A

(iii) $y = 10^x \Rightarrow$ Graph D

(iv) $y = (0.5)2^x \Rightarrow$ Graph C

Q2 $A(n) = 1000 \times 2^{0.2n}$

(i) $A(0) = 1000 \times 2^{0.2(0)} = 1000 \times 2^0 = 1000 \times 1 = 1000\,ha$

(ii) (a) 10 weeks

$A(10) = 1000 \times 2^{0.2(10)} = 1000 \times 2^{(2)} = 1000 \times 4 = 4000\,ha$

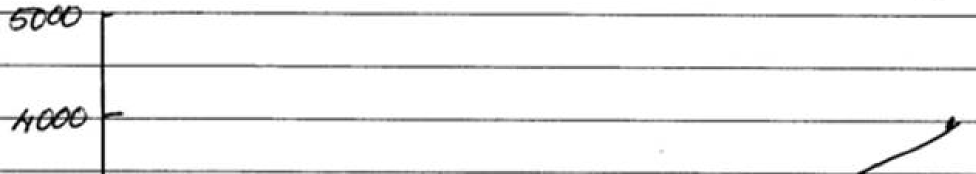
(b) 12 weeks

$A(12) = 1000 \times 2^{0.2(12)} = 1000 \times 2^{2.4} = 1000 \times (5.278) = 5278\,ha$

(iii)

n	0	2	4	6	8	10
$A(n)$	1000	1320	1741	2297	3031	4000

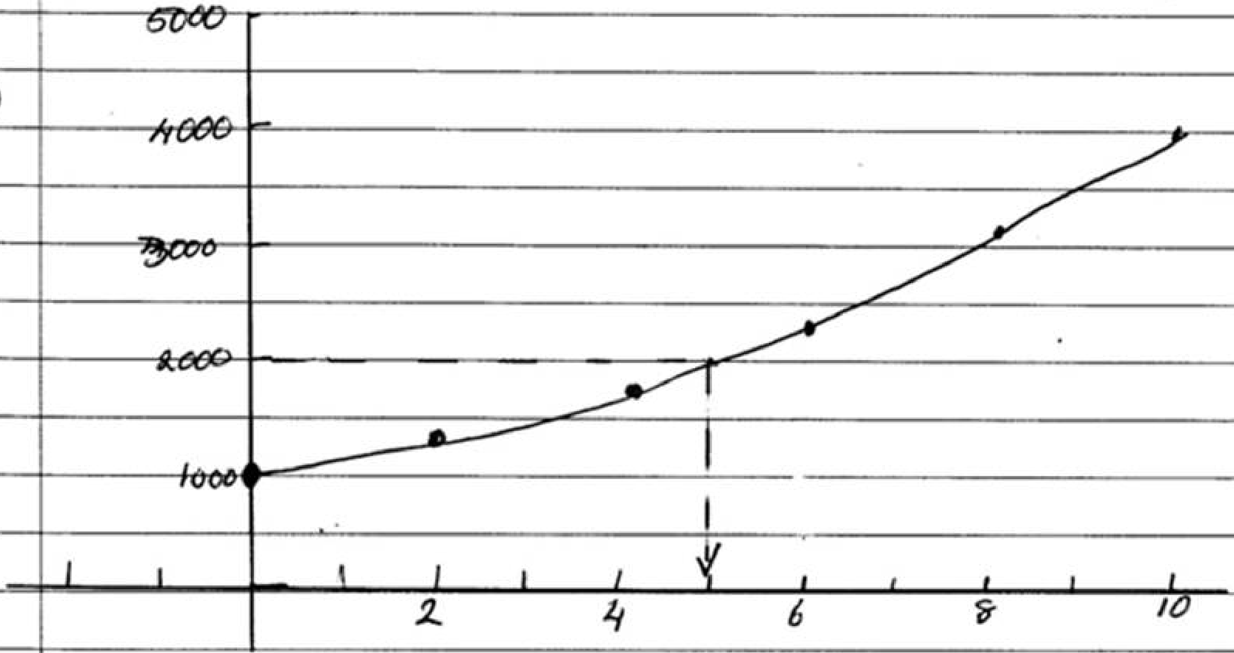
(calculate Table function)



(iii)

n	0	2	4	6	8	10
$A(n)$	1000	1320	1741	2297	3031	4000

(calculator Table function)



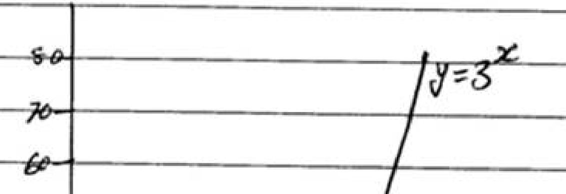
(iv) Doubling time ⇒ from 1000 to 2000 = 5 weeks

Q3 (i) Decreasing
(ii) Decreasing
(iii) Increasing
(iv) Decreasing

Q4 (i) 0.6 (ii) 3 (iii) 8 (iv) 6.

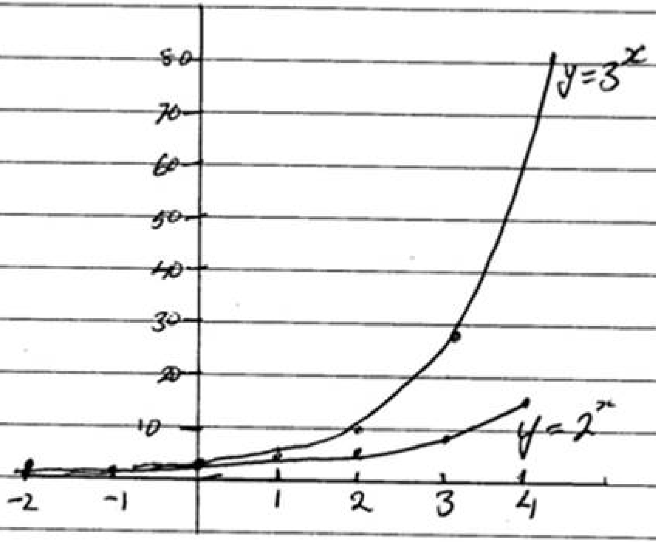
Q5 (i)

x	-2	-1	0	1	2	3	4
$y=2^x$	$1/4$	$1/2$	1	2	4	8	16
$y=3^x$	$1/9$	$1/3$	1	3	9	27	81



Q5 (i)

x	-2	-1	0	1	2	3	4
$y=2^x$	$1/4$	$1/2$	1	2	4	8	16
$y=3^x$	$1/9$	$1/3$	1	3	9	27	81



(iv) $-2 \leq x \leq 0$

(v) $0 < x \leq 4$

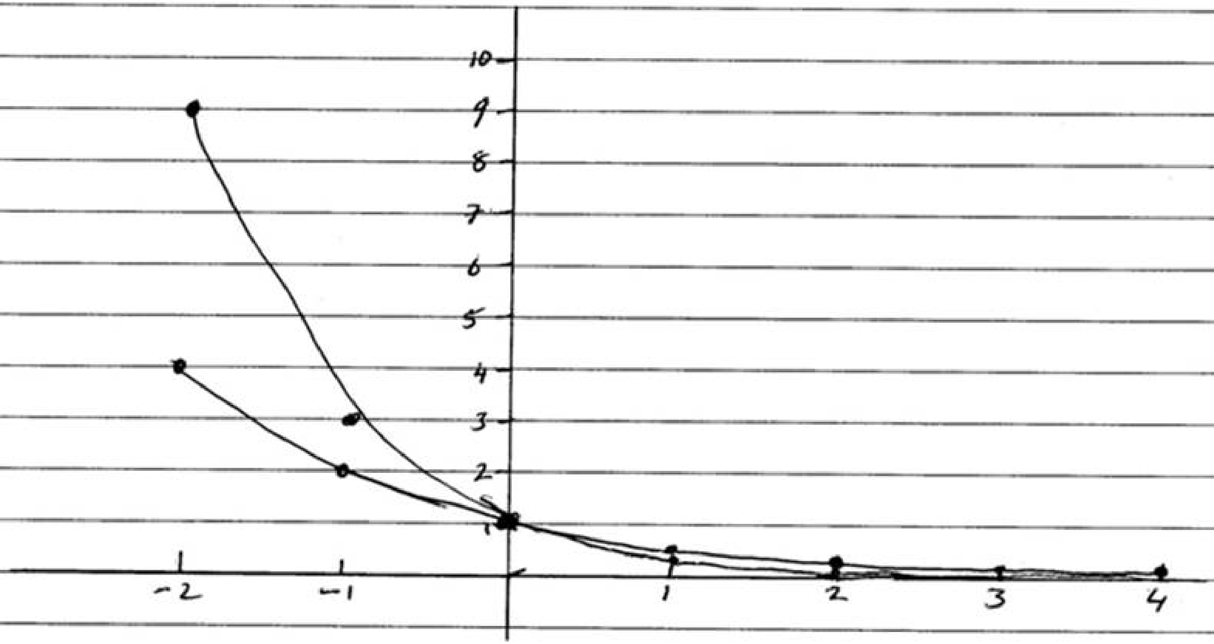
(vi) $x = 0.$

(ii)

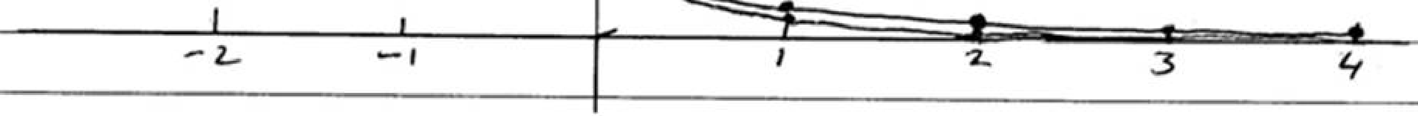
x	-2	-1	0	1	2	3	4
$y=2^{-x}$	4	2	1	$1/2$	$1/4$	$1/8$	$1/16$
$y=3^{-x}$	9	3	1	$1/3$	$1/9$	$1/27$	$1/81$

$\frac{1}{2^x}$

$\frac{1}{3^x}$



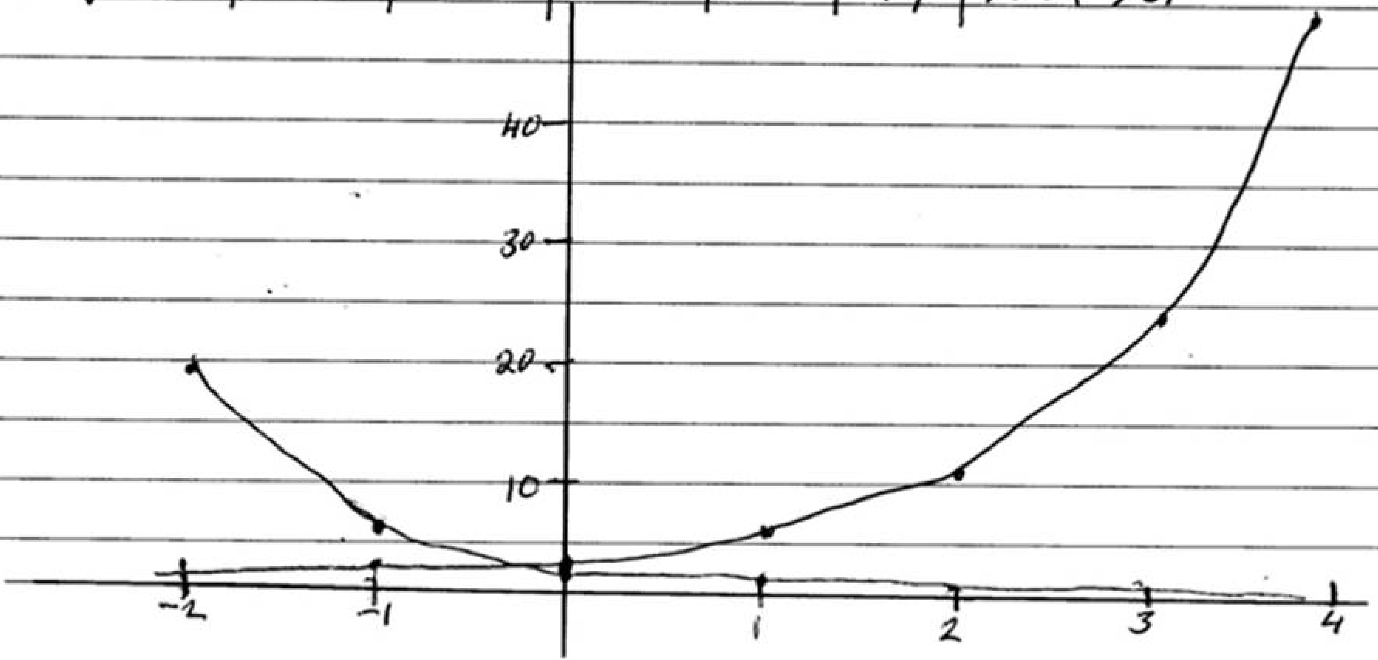
(vii) $0 < x \leq 4$



(vii) $0 < x \le 4$

(iii)

x	-2	-1	0	1	2	3	4
$y = 3 \cdot 2^{x}$	$3/4$	$3/2$	3	6	12	24	48
$y = 2 \cdot 3^{-x}$	18	6	2	$2/3$	$2/9$	$2/27$	$2/81$



Q6 $D = 18(0.72)^T$

(i) Decay as $a < 1 \Rightarrow$ graph is decreasing.

(ii) (a) $T = 5° \Rightarrow D = 18(0.72)^5 = 3.48 = 3$ days

(b) $T = 2° \Rightarrow D = 18(0.72)^2 = 9.33 = 9$ days

(c) $T = 0° \Rightarrow D = 18(0.72)^0 = 18$ days

(iii) At least 5 days $\Rightarrow \geq 5$

$$18(0.72)^T \geq 5$$

$$(0.72)^T \geq \frac{5}{18}$$

$$\log(0.72)^T \geq \log 5/18$$

$$T \log 0.72 \geq \log 5/18$$

$$T \geq \frac{\log 5/18}{\log 0.72}$$

$$T \geq 3.899$$

$\Rightarrow T = 3.9°C$

Q10 $E \cong 10^{1.5m+4.8}$

$m=7 \Rightarrow E \cong 10^{1.5(7)+4.8} = 10^{15.3}$

$m=5 \Rightarrow E \cong 10^{1.5(5)+4.8} = 10^{12.3}$

No of times greater $= \dfrac{10^{15.3}}{10^{12.3}}$ (subtract Indices)

$= 10^{3} = 1000.$

Q11

$A = P(1+i)^{t}$ $P=5000$ $i=0.0055.$

$A = 5000(1.0055)^{t}$

(i) $t=1 \Rightarrow A = 5000 \times (1.0055)^{1} = 5027.50$

(ii) $t=2 \Rightarrow A = 5000 \times (1.0055)^{2} = 5055.15$

(iii) $t=3 \Rightarrow A = 5000 \times (1.0055)^{3} = 5082.95$

(iv) $A = 5000(1.0055)^{t}$

Q2 $P(t) = 40b^t$

(i) $t = 0 \Rightarrow P(0) = 40b^0 = 40$

(ii) $t = 1 \Rightarrow 40b^1 = 48$

$b = \frac{48}{40}$

$b = 1.2 \Rightarrow$ Increasing N° of flies

(iii)

t	0	1	2	3	4	5
P_t	40	48	57.6	69.12	82.94	99.53.