

## Test Yourself 4

### A Questions

Q1 (i) radius = dis  $(-1, 5)$  to  $(1, 2)$   
 $= \sqrt{(1+1)^2 + (2-5)^2} = \sqrt{4+9} = \sqrt{13}$

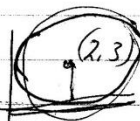
(ii) Egn  $(x+1)^2 + (y-5)^2 = 13$

Q2  $x^2 + y^2 - 2x - 4y - 9 = 0$   
centre =  $(1, 2)$   
radius =  $\sqrt{1^2 + 2^2 + 9} = \sqrt{14}$

eqn of circle: centre  $(0, 0)$  and  $r = \sqrt{14}$

$$x^2 + y^2 = 14$$

Q3 centre  $(2, 3)$  and touches  $x$ -axis  
 $\Rightarrow$  radius = 3



$\Rightarrow$  eqn:  $(x-2)^2 + (y-3)^2 = 9$  . .

Q4  $3x - 4y + 25 = 0$  and  $x^2 + y^2 = 25$

Tangent  $\Rightarrow$  only one pt of intersection.

$$3x - 4y + 25 = 0$$

$$3x = 4y - 25$$

$$x = \frac{4}{3}y - \frac{25}{3}$$

Sub into circle

$$x^2 + y^2 = 25$$

$$\left(\frac{4}{3}y - \frac{25}{3}\right)^2 + y^2 = 25$$

$$\frac{16}{9}y^2 - \frac{200}{9}y + \frac{625}{9} + y^2 = 25 \quad (\times 9)$$

$$16y^2 - 200y + 625 + 9y^2 = 225$$

$$25y^2 - 200y + 400 = 0 \quad (\div 25)$$

$$y^2 - 8y + 16 = 0$$

$$(y-4)(y-4) = 0$$

$$y = 4$$

only 1 y co-ord.

$$x = \frac{4}{3}y - \frac{25}{3}$$

$$x = \frac{16}{3} - \frac{25}{3}$$

$$x = -3$$

pt of n is (-3, 4)

only 1 pt  $\Rightarrow$  is a Tangent

Q5

(-1, -3) (3, 1) end pts of Diameter  
 $\Rightarrow$  centre = mid pt =  $\left(\frac{-1+3}{2}, \frac{-3+1}{2}\right) = (1, -1)$  centre

$$\text{radius} = \text{dis } (1, -1) \text{ to } (-1, -3)$$

$$= \sqrt{(1+1)^2 + (-1+3)^2} = \sqrt{4+4} = \sqrt{8}$$

eqn of circle: centre (1, -1) and  $r = \sqrt{8}$

$$(x-1)^2 + (y+1)^2 = 8$$

Q6  $(x-5)^2 + y^2 = 36$  centre  $(5,0)$   $r=6$

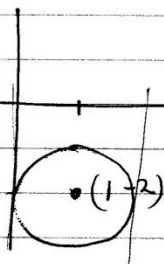
Meets  $x$ -axis  $\Rightarrow y=0$ .

$$\begin{aligned}(x-5)^2 &= 36 \\ x^2 - 10x + 25 &= 36 \\ x^2 - 10x - 11 &= 0 \\ (x-11)(x+1) &= 0 \\ x &= 11 \quad x = -1\end{aligned}$$

$P(11,0)$  and  $Q(-1,0)$

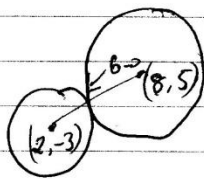
Q7  $x^2 + y^2 - 2x + 4y + 4 = 0$

centre  $(1, -2)$  radius  $= \sqrt{1^2 + (-2)^2 - 4} = 1$



2 Tangents: (1)  $x=0$   
(2)  $x=2$ .

Q8



Touch externally  $\Rightarrow r_1 + r_2 = \text{dis between 2 centres}$

$$\text{dis } (2, -3) \text{ to } (8, 5) = \sqrt{(8-2)^2 + (5+3)^2} = 10.$$

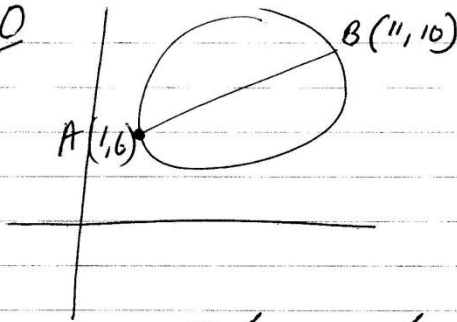
$r_1 = 6 \Rightarrow r_2 = 4$   
Ans radius of  $K_2 = 4$  units

● Q9  $(0,0)$  inside  $(x-5)^2 + (y+2)^2 = 30$

$$(0-5)^2 + (0+2)^2$$
$$25 + 4$$
$$29$$

$$29 < 30 \Rightarrow \text{inside}$$

● Q10



$A \rightarrow B$  is diameter

$\Rightarrow$  centre = midpt.

$$\left(\frac{1+11}{2}, \frac{6+10}{2}\right) = (6, 8) \text{ centre}$$

$$\text{radius} = \text{dps } (6, 8) \text{ to } (1, 6)$$
$$= \sqrt{(6-1)^2 + (8-6)^2} = \sqrt{29}$$

$\Rightarrow$  Eqn of Circle: Centre  $(6, 8)$   $r = \sqrt{29}$

$$(x-6)^2 + (y-8)^2 = 29$$

## B Questions

Q1  $x^2 + y^2 - 6x - 2y - 3 = 0$   
centre  $(3, 1)$   $r = \sqrt{3^2 + 1^2 + 3} = \sqrt{13}$

~~$(3, 1)$~~   $(5, 4)$

Eqn of Tangent:  $\perp$  to  $M$  of radius  
 $m$  radius =  $\frac{4-1}{5-3} = \frac{3}{2} \Rightarrow \perp M = -\frac{2}{3}$

eqn: pt  $(5, 4)$   $m = -1$   
 $y - 4 = -\frac{2}{3}(x - 5)$   
 $3y - 12 = -2x + 10$   
 $2x + 3y - 22 = 0$

Q2 (i) Show radius = 6  
 $\perp$  dis from  $(5, -1)$  to  $3x - 4y + 11 = 0$  is 6.

$$\frac{|3(5) - 4(-1) + 11|}{\sqrt{3^2 + 4^2}} = \frac{30}{5} = 6 \therefore r = 6.$$

(ii)  $x + py + 1 = 0$  is also a tangent.  
 $\Rightarrow \perp$  dis from  $(5, -1)$  to  $x + py + 1 = 0$  is 6.

$$\frac{|(5) + p(-1) + 1|}{\sqrt{1^2 + p^2}} = 6$$

$$\frac{|6 - p|}{\sqrt{1^2 + p^2}} = 6 \quad (\text{sq both sides})$$

$$\frac{36 - 12p + p^2}{1 + p^2} = 36$$

$$36 - 12p + p^2 = 36 + 36p^2$$

$$35p^2 + 12p = 0$$

$$p(35p + 12) = 0$$

$$\Rightarrow p = 0 \quad \text{or} \quad p = -12/35$$

Q3  $8x + 3y + k = 0$  Tangent to  $x^2 + y^2 + 4x - 3y - 12 = 0$   
 $\rightarrow$  Centre  $(-2, 3/2)$   
radius =  $\sqrt{(-2)^2 + (3/2)^2 + 12} = \frac{\sqrt{73}}{2}$

$\Rightarrow$   $\perp$  Dis from  $(-2, 3/2)$  to  $8x + 3y + k = 0$  is  $\frac{\sqrt{73}}{2}$

$$\frac{|8(-2) + 3(3/2) + k|}{\sqrt{8^2 + 3^2}} = \frac{\sqrt{73}}{2}$$

$$\frac{|-23/2 + k|}{\sqrt{73}} = \frac{\sqrt{73}}{2}$$

$$-\frac{23}{2} + k = \frac{1}{2} \quad \text{or} \quad -\frac{23}{2} + k = -\frac{1}{2}$$

$$k = \frac{1}{2} + \frac{23}{2}$$

$$k = \frac{24}{2} = 12$$

$$k = -1/2 + 23/2$$

$$k = 22/2 = 11$$

Q4 (i)  $(5, 2)$  on  $x^2 + y^2 + px - 2y + 5 = 0$   
 $(5)^2 + (2)^2 + p(5) - 2(2) + 5 = 0$   
 $25 + 4 + 5p - 4 + 5 = 0$   
 $5p = -30$   
 $p = -6$

(ii)  $x - y - 1 = 0$  intersects  $x^2 + y^2 - 6x - 2y + 5 = 0$   
 $x = (y + 1)$   
 $\Rightarrow (y + 1)^2 + y^2 - 6(y + 1) - 2y + 5 = 0$   
 $y^2 + 2y + 1 + y^2 - 6y - 6 - 2y + 5 = 0$   
 $2y^2 - 6y + 0 = 0$   
 $y(2y - 6) = 0$   
 $y = 0$  or  $y = 3$

find  $x$ :  $x = y + 1$   
at  $y = 0$   $x = 1$       at  $y = 3$   $x = 3 + 1$   
 $x = 4$

Pts of  $\cap$  are  $(1, 0)$  and  $(4, 3)$

Q5  $x^2 + y^2 + 2x - 2y - 23 = 0$  centre  $(-1, 1)$   
 $r_1 = \sqrt{(-1)^2 + (1)^2 + 23} = 5$   
 $x^2 + y^2 - 14x - 2y + 41 = 0$  centre  $(7, 1)$   
 $r_2 = \sqrt{7^2 + 1^2 - 41} = 3$

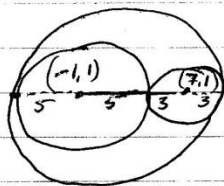
(i) Touch externally  $\Rightarrow r_1 + r_2 = \text{dis between centres}$

Dis between  $(-1, 1)$  and  $(7, 1)$   
 $\sqrt{(7+1)^2 + (1-1)^2} = 8$

$5 + 3 = 8$  True  $\therefore$  Touch externally

(ii) Touch internally  $\Rightarrow r_2 - r_1 = d$

Diameter of  $K = 16$  units  $\Rightarrow$  radius  $K = 8$  units.



centre is mid pt of  $(-1, 1)$   $(7, 1)$

$\Rightarrow \left(\frac{-1+7}{2}, \frac{1+1}{2}\right) = (3, 1)$

eqn of circle centre  $(3, 1)$  and  $r = 8$

$(x-3)^2 + (y-1)^2 = 64$



Q6  $x^2 + y^2 - 10y + 20 = 0$

(i)  $C = (0, 5)$

(ii)  $r = \sqrt{0^2 + 5^2 - 20} = \sqrt{5}$

(iii)  $y = 2x$  is a tangent.

$\Rightarrow 2x - y = 0$   $\perp$  dis from  $(0, 5)$  is  $\sqrt{5}$

$$\frac{|2(0) - (5)|}{\sqrt{2^2 + 1^2}} = \sqrt{5}$$

$$\frac{|-5|}{\sqrt{5}} = \sqrt{5}$$

$$|-5| = 5 \quad (\text{sq both sides})$$

$$25 = 25 \quad \text{True} \Rightarrow \text{is a tangent.}$$

$y = 2x$  and  $x^2 + y^2 - 10y + 20 = 0$

$$\Rightarrow x^2 + (2x)^2 - 10(2x) + 20 = 0$$

$$x^2 + 4x^2 - 20x + 20 = 0$$

$$5x^2 - 20x + 20 = 0 \quad (\div 5)$$

$$x^2 - 4x + 4 = 0$$

$$(x - 2)(x - 2) = 0$$

$$x = 2 \quad \text{only one pt.}$$

find  $y$ :  $y = 2x$

$$y = 2(2)$$

$$y = 4$$

pt of  $\cap$  is  $(2, 4)$

● Q7  $x^2 + y^2 = 4$  centre (0,0)  $r_1 = 2$

$x^2 + y^2 - 8x - 6y + 16 = 0$  centre (4,3)  
 $r_2 = \sqrt{4^2 + 3^2 - 16} = 3$

Common Tangent  $S_1 - S_2$

$$\begin{array}{r} \ominus x^2 + y^2 \oplus 4 = 0 \\ x^2 + y^2 - 8x - 6y + 16 = 0 \\ \hline -8x - 6y + 20 = 0 \quad (\div -2) \end{array}$$

● Egn of Tangent is:  $4x + 3y - 10 = 0$

Q8

(i) (2,5) on  $x^2 + y^2 + 2gx + 2fy + 7 = 0$   
 $(2)^2 + (5)^2 + 2g(2) + 2f(5) + 7 = 0$   
 $4 + 25 + 4g + 10f + 7 = 0$

$4g + 10f + 36 = 0 \quad (\div 2)$

$2g + 5f + 18 = 0$  ①

(-2,1) on  $x^2 + y^2 + 2gx + 2fy + 7 = 0$   
 $4 + 1 - 4g + 2f + 7 = 0$

$-4g + 2f + 12 = 0 \quad (\div 2)$

$2g - f - 6 = 0$  ②

(ii)  $2g + 5f = -18$

$2g - f = 6$

$6f = -24$

$f = -4$

find g:

$2g - f - 6 = 0$

$2g + 4 - 6 = 0$

$2g = 2$

$g = 1$

(iii) Eqn of Circle is:  $x^2 + y^2 + 2x - 8y + 7 = 0$

Centre  $(-1, 4)$   $r = \sqrt{(-1)^2 + 4^2 - 7} = \sqrt{10}$

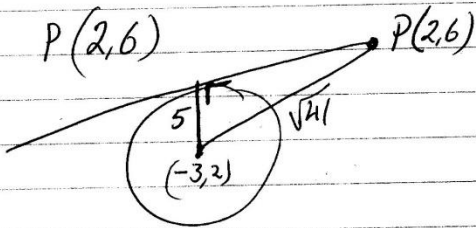
Q9  $(x+3)^2 + (y-2)^2 = 25$

(i) Centre  $(-3, 2)$

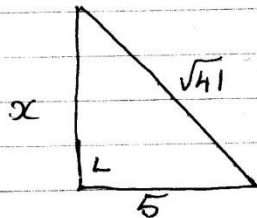
(ii) radius = 5

$N(0, -2)$  on  $(x+3)^2 + (y-2)^2 = 25$   
 $(0+3)^2 + (-2-2)^2$   
 $9 + 16$

$25 = 25$  True  $\Rightarrow$   $N$  on Circle



Dis  $(2, 6)$  to  $(-3, 2)$   
 $= \sqrt{(-3-2)^2 + (2-6)^2} = \sqrt{41}$



$$\sqrt{41}^2 = x^2 + 5^2$$

$$41 = x^2 + 25$$

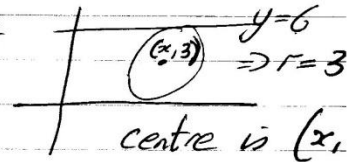
$$16 = x^2$$

$$\Rightarrow 4 = x$$

Length = 4 units.

Q10 centre on line  $x - 2y - 1 = 0$ .

$x$ -axis and  $y = 6$  are Tangents



centre is  $(x, 3)$

Find centre:  $(x, 3)$   $y = 3$  on line  $x - 2y - 1 = 0$   
 $x - 6 - 1 = 0$   
 $x = 7$

eqn of circle centre  $(7, 3)$  and  $r = 3$

Eqn of circle  $(x - 7)^2 + (y - 3)^2 = 9$

---

---

## C. Questions

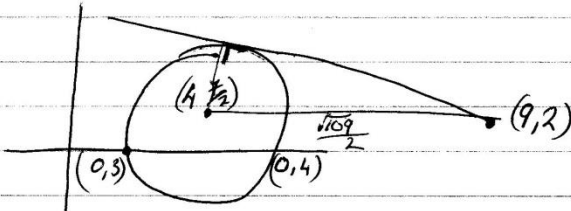
Q1 cuts  $y \Rightarrow x=0$  on  $x^2 + y^2 - 8x - 7y + 12 = 0$

$$y^2 - 7y + 12 = 0$$
$$(y - 3)(y - 4) = 0$$

$y = 3$      $y = 4$   
 $(0, 3)$  and  $(0, 4)$  are the intercepts

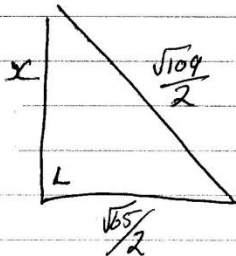
$\Rightarrow$  length = 1 unit.

$$x^2 + y^2 - 8x - 7y + 12 = 0 \quad \text{centre } (4, \frac{7}{2})$$
$$\text{radius} = \sqrt{4^2 + (\frac{7}{2})^2 - 12}$$
$$= \frac{\sqrt{65}}{2}$$



distance from  $(9, 2)$  to  $(4, \frac{7}{2})$

$$\sqrt{(9-4)^2 + (2-\frac{7}{2})^2} = \frac{\sqrt{109}}{2}$$



$$\left(\frac{\sqrt{109}}{2}\right)^2 = \left(\frac{\sqrt{65}}{2}\right)^2 + x^2$$

$$\frac{109}{4} - \frac{65}{4} = x^2$$

$$11 = x^2$$

$$\sqrt{11} = x$$

- Q2 line // to  $3x - 4y + 1 = 0$  is  $3x - 4y + C = 0$

$$x^2 + y^2 - 8x + 2y - 8 = 0, \text{ centre } (4, -1)$$

$$r = \sqrt{4^2 + (-1)^2 + 8} = 5$$

Tangents  $\Rightarrow$   $\perp$  dis from  $(4, -1)$  to  $3x - 4y + C = 0$  is 5

$$\frac{|3(4) - 4(-1) + C|}{\sqrt{3^2 + 4^2}} = 5$$

$$\frac{|16 + C|}{5} = 5$$

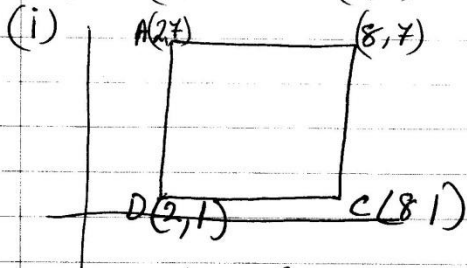
$$|16 + C| = 25 \Rightarrow 16 + C = 25 \text{ or } 16 + C = -25$$

$$C = 9 \text{ or } C = -41$$

$\Rightarrow$  2 Tangents are:  $3x - 4y + 9 = 0$  and  $3x - 4y - 41 = 0$

Q3 ABCD is a square  $\Rightarrow$  all sides equal in length.

dis  $(2, 7)$  to  $(8, 7)$  is 6 units.

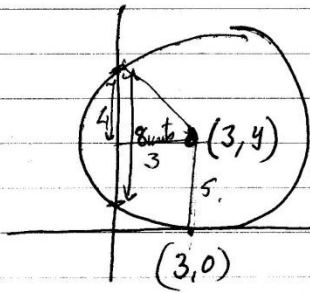


$\Rightarrow C(8, 1)$  and  $D(2, 1)$

(ii) Centre of circle is mid pt of a diagonal.  
 $(2, 7)$  and  $(8, 1)$  Mid pt =  $\left(\frac{2+8}{2}, \frac{7+1}{2}\right) = (5, 4)$

(iii) Inside the square  $\Rightarrow r = 3$   
 Eqn is:  $(x-5)^2 + (y-4)^2 = 9$

Q4



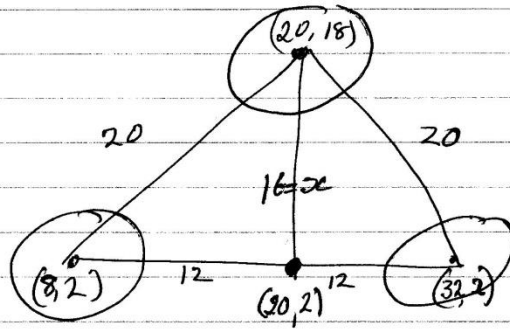
$\Rightarrow$  radius = 5.

$\Rightarrow$  centre is (3, 5)

eqn of circle:  $(x-3)^2 + (y-5)^2 = 25$

Q5

$(x-20)^2 + (y-18)^2 = 16 \Rightarrow$  centre (20, 18)  $r=4$



$$20^2 = 12^2 + x^2$$

$$400 - 144 = x^2$$

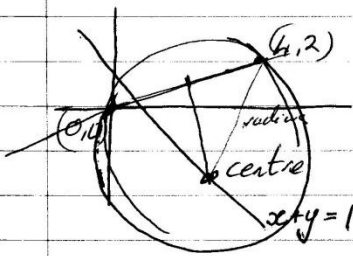
$$256 = x^2$$

$$16 = x$$

Eqns are:  $(x-8)^2 + (y-2)^2 = 16$   
and  $(x-32)^2 + (y-2)^2 = 16$

Q6 Eqn of circle: require centre and radius

centre is on the line  $x+y=1$ , It is also on the  $\perp$  bisector of chord  $(0,0)$  to  $(4,2)$



$(0,0), (4,2)$  mid pt  $(2,1)$   
slope  $= \frac{2}{4} = \frac{1}{2} \Rightarrow \perp m = -2$

eqn of  $\perp$  bisector:

$$y-1 = -2(x-2)$$

$$y-1 = -2x+4$$

$$y+2x = 5$$

Centre is the point of intersection of  $x+y=1$  and  $y+2x=5$

$$\begin{array}{r} \ominus x+y=1 \\ 2x+y=5 \\ \hline x=4 \end{array}$$

$$\begin{array}{r} \text{find } y: x+y=1 \\ 4+y=1 \\ y=-3 \end{array}$$

centre is  $(4, -3)$

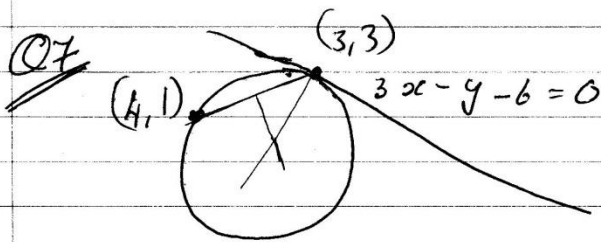
Radius is dis from  $(4, -3)$  to  $(0,0)$

$$= \sqrt{(4-0)^2 + (-3-0)^2} = 5$$

Eqn of circle centre  $(4, -3)$  and  $r=5$

$$(x-4)^2 + (y+3)^2 = 25$$





Centre is on the  $\perp$  bisector of chord  $(4,1)$   $(3,3)$   
 also on line  $\perp$  to  $3x - y - 6 = 0$  at  $(3,3)$

$\perp$  Bisector of Chord: Mid pt =  $(\frac{7}{2}, 2)$   
 slope =  $\frac{3-1}{3-4} = \frac{2}{-1} = -2 \Rightarrow \perp m = \frac{1}{2}$

$$\begin{aligned} \text{Eqn: } y - 2 &= \frac{1}{2}(x - \frac{7}{2}) \\ 2y - 4 &= x - \frac{7}{2} \\ 4y - 8 &= 2x - 7 \\ \boxed{2x - 4y + 1} &= 0 \end{aligned}$$

line  $\perp$  to  $3x - y - 6 = 0$  at  $(3,3)$   
 $m = -3/-1 = 3 \Rightarrow \perp m = -\frac{1}{3}$

$$\begin{aligned} \text{Eqn: } y - 3 &= -\frac{1}{3}(x - 3) \\ 3y - 9 &= -x + 3 \\ \boxed{x + 3y - 12} &= 0 \end{aligned}$$

Centre is pt of  $\cap$  of 2 lines

$$\begin{aligned} 2x - 4y + 1 &= 0 \quad (x-1) \\ x + 3y - 12 &= 0 \quad (x-2) \\ \hline -2x + 4y - 1 &= 0 \\ x + 3y - 12 &= 0 \\ \hline 10y &= 25 \\ y &= \frac{5}{2} \end{aligned}$$

find x:

$$2x - 4(\frac{5}{2}) + 1 = 0$$

$$\begin{aligned} 2x - 9 &= 0 \\ x &= \frac{9}{2} \end{aligned}$$

centre is  $(\frac{9}{2}, \frac{5}{2})$

- radius is distance from  $(\frac{9}{2}, \frac{5}{2})$  and  $(3, 3)$

$$\sqrt{(\frac{9}{2}-3)^2 + (\frac{5}{2}-3)^2} = \frac{\sqrt{10}}{2}$$

Egn of Circle: Centre  $(\frac{9}{2}, \frac{5}{2})$   $r = \frac{\sqrt{10}}{2}$

$$(x - \frac{9}{2})^2 + (y - \frac{5}{2})^2 = \frac{10}{4}$$

- OR <sup>Egn of Circle</sup> Centre =  $(\frac{9}{2}, \frac{5}{2})$   $r = \frac{\sqrt{10}}{2}$   
(-g, -f)

$$r = \sqrt{(\frac{9}{2})^2 + (\frac{5}{2})^2 - c} = \frac{\sqrt{10}}{2}$$

$$\sqrt{\frac{53}{2} - c} = \frac{\sqrt{10}}{2}$$

$$\frac{53}{2} - c = \frac{10}{4} \quad (\times 4)$$

$$106 - 4c = 10$$

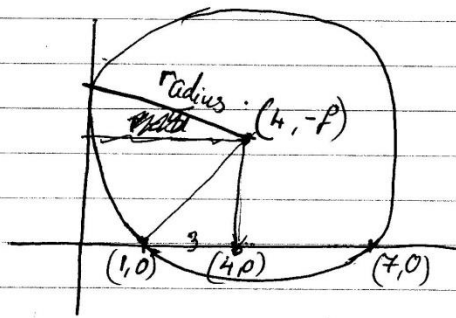
$$96 = 4c$$

$$24 = c$$

$\Rightarrow$  Egn of Circle  $x^2 + y^2 + 2gx + 2fy + c = 0$   
 $x^2 + y^2 - 9x - 5y + 24 = 0$

---

Q8



Centre  $(-g, -f)$

diagram  $-g = 4$

$$\Rightarrow \boxed{g = -4}$$

eqn of circle:  $x^2 + y^2 + 2gx + 2fy + C = 0$

$$\left. \begin{array}{l} (1,0) \Rightarrow 1 + 2g + C = 0 \\ (7,0) \Rightarrow 49 + 14g + C = 0 \end{array} \right\} \text{ use to find } C$$

$$2g + C = -1 \quad \text{but } g = -4$$

$$-8 + C = -1$$

$$\boxed{C = 7}$$

~~But~~ need to find  $f$ : Touches  $y$ -axis  $\Rightarrow f^2 = C$

$$\text{radius} = \sqrt{g^2 + f^2 - C} \quad \text{but } g = -4 \text{ and } C = 7$$

$$= \sqrt{(-4)^2 + f^2 - 7} \quad \text{but } f^2 = C = 7$$

$$\sqrt{(-4)^2 + 7 - 7} = \sqrt{16} = \boxed{4 = \text{radius}}$$

$$\text{i.e. } \sqrt{g^2 + f^2 - C} = 4$$

$$\sqrt{(-4)^2 + f^2 - 7} = 4$$

$$\sqrt{9 + f^2} = 4$$

$$9 + f^2 = 16$$

$$f^2 = 7$$

$$f = \pm\sqrt{7}$$

hence Centre  $(4, \sqrt{7})$   
radius = 4

$$\text{eqn: } (x-4)^2 + (y-\sqrt{7})^2 = 16$$

Q9(i)  $x^2 + y^2 + 2gx + 2fy + c = 0$

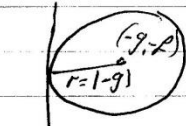
$$r = \sqrt{g^2 + f^2} = c$$

$$\text{but } r = |g|$$

$$\therefore |g| = \sqrt{g^2 + f^2} = c \quad (\text{sq both sides})$$

$$g^2 = g^2 + f^2 - c^2$$

$$c = f^2 \quad \text{QED.}$$



Touches y axis  
 $\Rightarrow r = |g|$

(ii)  $x^2 + y^2 + 2gx + 2fy + c = 0$

$$(-3, 6) \Rightarrow 9 + 36 - 6g + 12f + c = 0 \Rightarrow -6g + 12f + c = -45 \quad \textcircled{1}$$

$$(-6, 3) \Rightarrow 36 + 9 - 12g + 6f + c = 0 \Rightarrow -12g + 6f + c = -45 \quad \textcircled{2}$$

Use ① and ② to eliminate g

$$\textcircled{1} \times 2 \quad 12g - 24f - 2c = 90$$

$$\textcircled{2} \quad -12g + 6f + c = -45$$

$$\hline -18f - c = 45$$

$$\text{but } c = f^2 \Rightarrow -18f - f^2 = 45$$

$$f^2 + 18f + 45 = 0$$

$$(f + 3)(f + 15) = 0$$

$$f = -3 \quad \text{and} \quad f = -15$$

$$f = -3$$

$$\Rightarrow c = (-3)^2 = 9$$

$$c = 9$$

$$f = -15 \Rightarrow c = (-15)^2$$

$$c = 225$$

find g:  $-6g + 12f + c = -45$

$$f = -3, c = 9 \Rightarrow -6g + 12(-3) + 9 = -45$$

$$-6g = -18$$

$$g = +3$$

$$f = -15, c = 225 \Rightarrow -6g + 12(-15) + 225 = -45$$

$$-6g = -90$$

$$g = 15$$

circle ①  $g = +3, f = -3, c = 9.$

$$\therefore \text{Eqn: } x^2 + y^2 + 6x - 6y + 9 = 0$$

Circle ②  $g = 15 \quad f = -15 \quad c = 225$

$$\therefore \text{Eqn: } x^2 + y^2 + 30x - 30y + 225 = 0$$

● Q10  $x^2 + y^2 - 2ax - 2by + b^2 = 0$

(i) centre  $(a, b)$

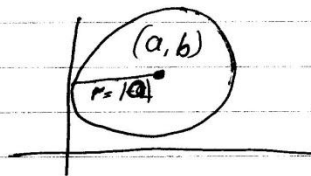
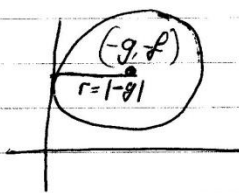
$$r = \sqrt{a^2 + b^2 - b^2} = \sqrt{a^2} \Rightarrow \underline{r = a}$$

(ii)  $r = \sqrt{g^2 + f^2 - c}$

$$a = \sqrt{a^2 + b^2 - b^2}$$

$$a = \sqrt{a^2}$$

$$a = a \quad \text{True } \del{a=a}$$



$r = a \Rightarrow$  Touches the y-axis.

(iii)  $x^2 + y^2 + 2gx + 2fy + c = 0$

$(1, 2) \Rightarrow 1 + 4 + 2g + 4f + c = 0 \Rightarrow \boxed{2g + 4f + c = -5} \quad (x2)$

$(2, 3) \Rightarrow 4 + 9 + 4g + 6f + c = 0 \Rightarrow \boxed{4g + 6f + c = -13} \quad (x-1)$

Touches y axis  $\Rightarrow \boxed{c = f^2}$

● eliminate  $g$  from ① and ②

$$4g + 8f + 2c = -10$$

$$-4g - 6f - c = 13$$

$$2f + c = 3$$

but  $c = f^2 \Rightarrow 2f + f^2 = 3$

$$f^2 + 2f - 3 = 0$$

$$(f + 3)(f - 1) = 0$$

$$\boxed{f = -3} \quad \boxed{f = 1}$$

$$f = 1 \Rightarrow \boxed{c = 1}$$

$$f = -3 \Rightarrow \boxed{c = 9}$$

Find  $g$ :  $2g + 4f + c = -5$

●  $f = -3, c = 9$ :  $2g + 4(-3) + 9 = -5$

$$2g = -2$$

$$\boxed{g = -1}$$

$$f=1, c=1, \Rightarrow 2g + 4(1) + 1 = -5$$
$$2g = -10$$
$$g = -5.$$

$$\text{Circle ①: } g = -1 \quad f = -3 \quad c = 9$$

$$\text{eqn: } x^2 + y^2 - 2x - 6y + 9 = 0$$

$$\text{Circle ②: } g = -5 \quad f = 1 \quad c = 1$$

$$\text{eqn: } x^2 + y^2 - 10x + 2y + 1 = 0$$

$$\text{Centre ① } (1, +3)$$

$$\text{Centre ② } (5, -1)$$

$$\text{Distance} = \sqrt{(5-1)^2 + (-1+3)^2}$$
$$= \sqrt{16+4}$$
$$= \sqrt{20} = 2\sqrt{5}$$