Revision Exercise (Advanced)

Q1 $2x^2 - 4x - 5$

$2(x^2 - 2x - 5/2)$

$2(x^2 - 2x + 1 - 1 - 5/2)$

$2((x-1)^2 - 7/2)$

$2(x-1)^2 - 7$.

(ii) $\Rightarrow$ Min Pt $(1, -7)$

(i) $2(x-1)^2 - 7 = 0$

$2(x-1)^2 = 7$

$(x-1)^2 = 7/2$

$x - 1 = \pm\sqrt{7/2}$

$x = 1 \pm \sqrt{7/2}$

Q2 $(2\sqrt{2} - \sqrt{3})^2$ Sq 1st + Twice product + Sq last.

$(2\sqrt{2})^2 - 2(2\sqrt{2})(\sqrt{3}) + (-\sqrt{3})^2$

$8 - 4\sqrt{6} + 3$

$11 - 4\sqrt{6}$

Q3 $\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{80}+\sqrt{5}} = \dfrac{\sqrt{7}+\sqrt{5}}{4\sqrt{5}+\sqrt{5}} = \dfrac{\sqrt{7}+\sqrt{5}}{5\sqrt{5}}$

$\dfrac{\sqrt{7}+\sqrt{5}}{5\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{35}+5}{25}$

Q4 $\sqrt{x+2} = x-4$ sq both sides

$x+2 = x^2 - 8x + 16$

$0 = x^2 - 9x + 14$

$0 = (x-2)(x-7)$

$x = 2 \quad x = 7$

Check $\sqrt{2+2} = 2-4$ $\quad \sqrt{7+2} = 7-4$

$\sqrt{4} = -2$ false $\quad \sqrt{9} = 3$ True.

$\Rightarrow$ Ans $= 7$

Q5 $8t^2 + 4t = S$

(i) $S = 10$ estimate = less than 1.

(ii) $8t^2 + 4t - 10 = 0$ ($\div$ across by 2)

$4t^2 + 2t - 5 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 4(4)(-5)}}{2(4)}$$

$$= \frac{-2 \pm \sqrt{84}}{8} \quad \nearrow 0.90 \quad \searrow -1.40$$

Ans is 0.90 as time cannot be negative

(iii) Actual Ans $= \frac{-2+\sqrt{84}}{8}$ rounded off $= 0.90$

$$\% \text{ Error} = \frac{\text{Error}}{\text{actual}} \times 100$$

$$= \frac{0.90 - \frac{-2+\sqrt{84}}{8}}{\frac{-2+\sqrt{84}}{8}} \times 100 = 0.486\%$$

Q6

$$\sigma = \frac{\sqrt{p(1+p)}}{n}$$

$$n\sigma = \sqrt{p(1+p)}$$

$$(n\sigma)^2 = p + p^2$$

$$0 = p^2 + p - (n\sigma)^2$$

$$p = \frac{-1 \pm \sqrt{1 - 4(1)(-n\sigma)^2}}{2}$$

$$p = \frac{-1 \pm \sqrt{1 + 4n^2\sigma^2}}{2}$$

Q9 $\quad -x^2 + 5x + 3 = x^2 + 5x - 1$

$$0 = 2x^2 - 4$$

$$x = \frac{0 \pm \sqrt{0 - 4(2)(-4)}}{2(2)}$$

$$x = \frac{\pm\sqrt{32}}{4}$$

$$x = \frac{\pm 4\sqrt{2}}{4} = \pm\sqrt{2}$$ Sols for x now find y

$y = x^2 + 5x - 1$
$y = (\sqrt{2})^2 + 5\sqrt{2} - 1$
$y = 2 + 5\sqrt{2} - 1$
$y = 1 + 5\sqrt{2}$

$(\sqrt{2},\ 1 + 5\sqrt{2})$

$y = x^2 + 5x - 1$
$y = (-\sqrt{2})^2 + 5(-\sqrt{2}) - 1$
$y = 2 - 5\sqrt{2} - 1$
$y = 1 - 5\sqrt{2}$

$(-\sqrt{2},\ 1 - 5\sqrt{2})$

Q10 $kx^2 - 2kx - 3k - 12 = 0$ real roots

$\Rightarrow b^2 - 4ac \geq 0$

$a = k, \quad b = -2k, \quad c = -3k - 12$

$(-2k)^2 - 4(k)(-3k-12) \geq 0$

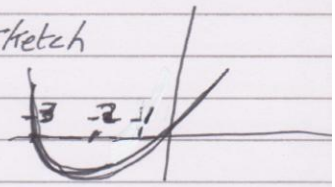
$4k^2 + 12k^2 + 48k \geq 0$

$16k^2 + 48k \geq 0$

$16k(k+3) \geq 0$

$k \geq 0 \qquad k \geq -3$

sketch



$-3 \geq k > 0$

Q11 $x^2 - \sqrt{3}x - 6 = 0$

$x^2 - (\text{Sum } x) + \text{product} = 0$

$\Rightarrow r_1 r_2 = -6.$

Q12

$3x + y = -1$

$x^2 + y^2 = 53$

$y = -3x - 1$

$x^2 + (-3x-1)^2 = 53$

$x^2 + 9x^2 + 6x + 1 = 53$

$10x^2 + 6x - 52 = 0$ (÷ across by 2)

$5x^2 + 3x - 26 = 0$

$(5x + 13)(x - 2) = 0$

$5x = -13$ $\quad x = 2.$

$x = -13/5$

Solve for y: $\quad y = -3x - 1$

at $x = -13/5$

$y = -3(-13/5) - 1$

$y = 34/5$

$(-13/5, 34/5)$

at $x = 2$

$y = -3(2) - 1$

$y = -6 - 1$

$y = -7$

$(2, -7)$

Q15 $(-2,-1)$ $(1,2)$ $(3,-16)$

Quadratic $\Rightarrow f(x) = ax^2+bx+c$.

$(-2,-1)$ $a(-2)^2+b(-2)+c = -1$

$\boxed{4a-2b+c=-1}$ (A)

$(1,2)$ $a(1)^2+b(1)+c = 2$

$\boxed{a+b+c=2}$ (B)

$(3,-16)$ $a(3)^2+b(3)+c = -16$

$\boxed{9a+3b+c=-16}$ (C)

(A) $4a-2b+c=-1$

$-$(B) $-a-b-c=-2$

$3a-3b=-3$

$\boxed{a-b=-1}$

(C) $9a+3b+c=-16$

$-$(B) $-a-b-c=-2$

$8a+2b=-18$

$\boxed{4a+b=-9}$

$a-b=-1$

$4a+b=-9$

$5a=-10$

$a=-2$

$a-b=-1$

$-2-b=-1$

$-b=1$

$b=-1$

$a+b+c=2$

$-2-1+c=2$

$c=5$

Quadratic is $f(x) = -2x^2-x+5$.