Ex 1.4

Q1

$$y = x - 4$$
$$y + 4 = x$$
$$\Rightarrow f^{-1}(x) = x + 4$$

Q2

$$y = 2x - 3$$
$$\frac{y+3}{2} = x$$
$$\Rightarrow f^{-1}(x) = \frac{x+3}{2}$$

Q3

$$y = 5x + 3$$
$$\frac{y-3}{5} = x$$
$$\Rightarrow f^{-1}(x) = \frac{x-3}{5}$$

Q4

$$y = 3x$$
$$\frac{y}{3} = x$$
$$\Rightarrow f^{-1}(x) = \frac{x}{3}$$

Q5

$$y = \frac{2x}{5}$$
$$\frac{5y}{2} = x$$
$$\Rightarrow f^{-1}(x) = \frac{5x}{2}$$

Q6

$$y = \frac{4x-3}{2}$$
$$\frac{2y+3}{4} = x$$
$$\Rightarrow f^{-1}(x) = \frac{2x+3}{4}$$

Q7 $y = \dfrac{x-6}{x}$

$yx - x = -6$

$x(y-1) = -6$

$x = \dfrac{-6}{y-1}$

$\Rightarrow f^{-1}(x) = \dfrac{-6}{x-1}$

Q8 $y = \dfrac{3x}{x-1}$

$y(x-1) = 3x$

$yx - y = 3x$

$yx - 3x = y$

$x(y-3) = y$

$x = \dfrac{y}{y-3}$

$\therefore f^{-1}(x) = \dfrac{x}{x-3}$

Q9 $y = \dfrac{10-2x}{3}$

$3y = 10 - 2x$

$2x = 10 - 3y$

$x = \dfrac{10-3y}{2}$

$\therefore f^{-1}(x) = \dfrac{10-3x}{2}$

Q11 $y = \frac{x}{3} - 2$

$3(y+2) = x$

$\Rightarrow f^{-1}(x) = 3(x+2)$.

Show $ff^{-1}(x) = x$

$f^{-1}(x) = 3(x+2)$

$f[3(x+2)] = \frac{\cancel{3}(x+2)}{\cancel{3}} - 2 = x$.

Q 12

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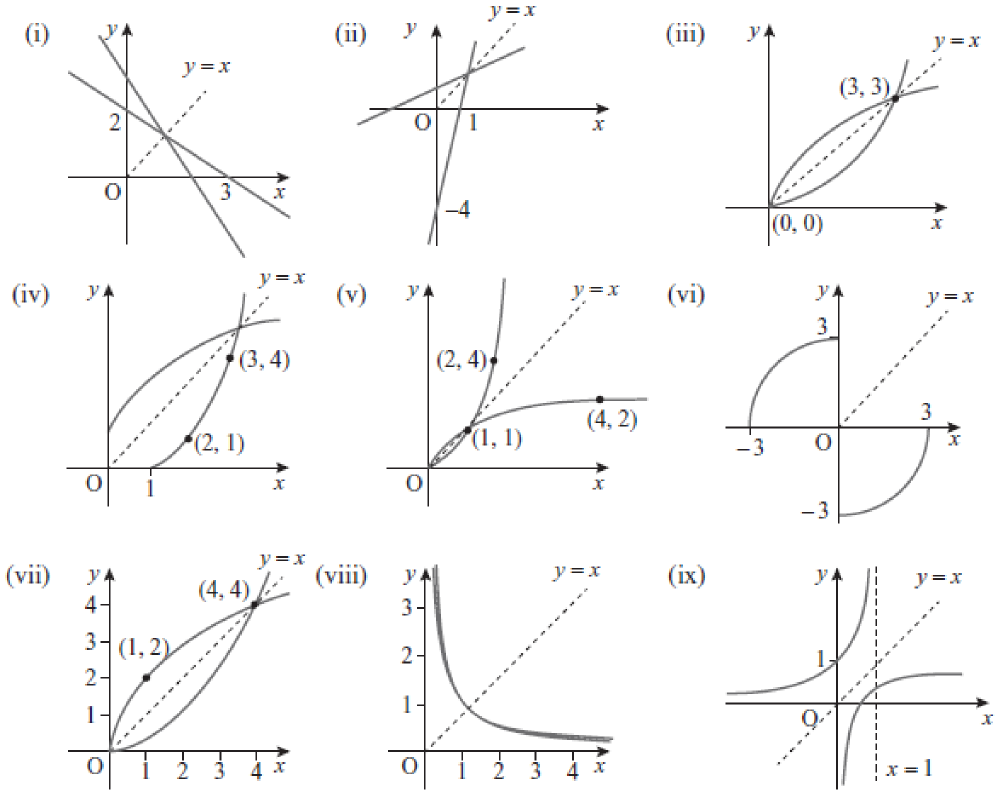
Q12.


(i)
y = x
2
O
3
(ii)
y = x
O
1
−4
(iii)
y = x
(3, 3)
(0, 0)
(iv)
y = x
(3, 4)
(2, 1)
O
1
(v)
y = x
(2, 4)
(1, 1)
(4, 2)
O
(vi)
y = x
3
−3
O
3
−3
(vii)
y = x
(4, 4)
(1, 2)
4
3
2
1
O
1
2
3
4
(viii)
y = x
3
2
1
O
1
2
3
4
(ix)
y = x
1
O
x = 1

Q14 $g(x) = \frac{1}{x-2}$

$y = \frac{1}{x-2}.$

$x-2 = \frac{1}{y}$

$x = \frac{1}{y} + 2.$

$\Rightarrow f^{-1}(x) = \frac{1}{x} + 2 \quad = \quad \frac{1+2x}{x} \quad \Rightarrow \quad k = 2.$

Q15 $f(x) = 2x-3 \qquad g(x) = x-4$

(i) $gf(x)$

$f(x) = 2x-3$

$g(2x-3) = 2x-3-4 \quad = 2x-7.$

$[gf(x)]^{-1}$

$y = 2x-7$

$\frac{y+7}{2} = x$

$[gf(x)]^{-1} = \frac{x+7}{2}$

(ii) $f^{-1}(x) = \frac{x+3}{2} \qquad g^{-1}(x) = x+4.$

$f^{-1}g^{-1}(x)$

$g^{-1}(x) = x+4$

$f^{-1}(x+4) = \frac{(x+4)+3}{2} \quad = \quad \frac{x+7}{2}.$

$\therefore \quad [gf(x)]^{-1} = f^{-1}g^{-1}(x).$ is True.

Q17 (i) $y = x^2+4x-6$

$y = x^2+4x+4-4-6$ complete Sq

$y = (x+2)^2-10$

$\sqrt{y+10}-2 = x$

$\Rightarrow f^{-1}(x) = \sqrt{x+10}-2 \qquad x \geqslant -10$

(ii) $f(x) = x^2-2x-5$

$y = x^2-2x+1-1-5$

$y = (x-1)^2-6$

$\sqrt{y+6}+1 = x$

$\Rightarrow f^{-1}(x) = 1+\sqrt{x+6} \qquad x \geqslant -6$

(iii) $y = x^2-8x-3$

$y = x^2-8x+16-16-3$

$y = (x-4)^2-19$

$\sqrt{y+19}+4 = x$

$\Rightarrow f^{-1}(x) = 4+\sqrt{x+19} \qquad x \geqslant -19$

(v) $y = x^2+8x+20$

$y = x^2+8x+16-16+20$

$y = (x+4)^2+4$

$\sqrt{y-4}-4 = x$

$\Rightarrow f^{-1}(x) = \sqrt{x-4}-4 \qquad x \geqslant 4$

Q18 $\quad f(x) = \dfrac{3-x}{2}, \quad -1 \le x \le 4.$

2 points are $(-1, 2)$ and $(4, -\tfrac{1}{2})$

$f^{-1}(x)$: $\quad y = \dfrac{3-x}{2}$

$$2y = 3 - x$$
$$x = 3 - 2y \Rightarrow f^{-1}(x) = 3 - 2x.$$

Domain of $f^{-1}(x)$ = Range of $f(x)$

$\Rightarrow -\tfrac{1}{2} \le x \le 2.$

2 points are $(-\tfrac{1}{2}, 4)$ and $(2, -1)$.

Domain $f^{-1}(x) = [-\tfrac{1}{2}, 2]$ and Range $f^{-1}(x) = [-1, 4]$

Q19 $\quad f : A \to R,\ f(x) = \sqrt{3-x}.$

$f(x)$ is defined for $\sqrt{3-x}$ positive

$\Rightarrow A = [-\infty, 3] \qquad x < 3.$

Q20 $\quad g : [b, 2] \to R \qquad g(x) = 1 - x^2$

$-x^2$ $\cap$ $\Rightarrow$ only use $\tfrac{1}{2}$ of graph $\Rightarrow b = 0$

$g^{-1}(x)$: $\quad y = 1 - x^2$

$$x^2 = 1 - y$$
$$x = \sqrt{1-y}$$

$\Rightarrow \quad g^{-1}(x) = \sqrt{1-x} \qquad x \le 1$