

Ex 2.10

Q1 (i) roots: $x = -1$, $x = 1$, $x = 3$
factors: $(x+1)(x-1)(x-3)$

$$\Rightarrow f(x) = a(x+1)(x-1)(x-3)$$

check for integer factor using $(0,3)$

$$3 = a(1)(-1)(-3)$$

$$3 = 3a$$

$$1 = a$$

$$\begin{aligned}\Rightarrow f(x) &= (x+1)(x-1)(x-3) \\ &= (x^2-1)(x-3) \\ &= x^3 - 3x^2 - x + 3\end{aligned}$$

(ii) roots: $x = -4$, $x = 1$, $x = 2$
factors: $(x+4)(x-1)(x-2)$

$$f(x) = a(x+4)(x-1)(x-2)$$

check for integer factor using $(0,8)$

$$8 = a(4)(-1)(-2)$$

$$8 = 8a$$

$$1 = a$$

$$\begin{aligned}f(x) &= (x+4)(x-1)(x-2) \\ &= (x^2 - x + 4x - 4)(x-2) \\ &= (x^2 + 3x - 4)(x-2) \\ &= x^3 - 2x^2 + 3x^2 - 6x - 4x + 8 \\ &= x^3 + x^2 - 10x + 8.\end{aligned}$$

Q2 (i) Green
Roots: $x = -3, x = 0, x = 2$.
Factors: $(x+3)(x)(x-2)$

Green: $f(x) = a(x+3)(x)(x-2)$ Test $(1, -4)$
 $-4 = a(4)(1)(-1)$
 $-4 = -4a$
 $1 = a$

$$\begin{aligned} f(x) &= (x+3)(x)(x-2) \\ &= (x^2+3x)(x-2) \\ &= x^3 - 2x^2 + 3x^2 - 6x \\ &= x^3 + x^2 - 6x \end{aligned}$$

Blue: $f(x) = a(x+3)(x)(x-2)$ Test $(1, -12)$
 $-12 = a(4)(1)(-1)$
 $-12 = a(-4)$
 $3 = a$

$$\begin{aligned} f(x) &= 3(x+3)(x)(x-2) \\ &= (3x^2+9x)(x-2) \\ &= 3x^3 - 6x^2 + 9x^2 - 18x \\ &= 3x^3 + 3x^2 - 18x \end{aligned}$$

Examining Graph $(1, -4)$
 $(1, -12)$

↓
Is amplified 3 times
→ ~~large~~ factor is 3

Q2 (ii) Roots: $x=1$ $x=2$ $x=3$
RED: $f(x) = a(x-1)(x-2)(x-3)$ Test (0,6)

$$6 = a(-1)(-2)(-3)$$

$$6 = -6a$$

$$-1 = a$$

$$\begin{aligned} f(x) &= -1(x-1)(x-2)(x-3) \\ &= -1(x^2-3x+2)(x-3) \\ &= -1(x^3-3x^2-3x^2+9x+2x-6) \\ &= -x^3+6x^2-11x+6 \end{aligned}$$

Blue: $f(x) = a(x-1)(x-2)(x-3)$ Test (0,12)

$$12 = a(-1)(-2)(-3)$$

$$12 = -6a$$

$$-2 = a$$

→ also found comparing
(0,6) to (0,12)

$$f(x) = -2x^3 + 12x^2 - 22x + 12$$

Q3

$$f(x) = ax^3 + bx^2 + cx + d$$

$$x=1 \quad x=-2 \quad x=\frac{1}{2}$$

$$f(x) = a(x-1)(x+2)(2x-1) \quad \text{Test (0,6)}$$

$$6 = a(-1)(2)(-1)$$

$$6 = 2a$$

$$3 = a$$

$$\begin{aligned} f(x) &= 3(x-1)(x+2)(2x-1) \\ &= (3x-3)(x+2)(2x-1) \\ &= (3x^2+6x-3x-6)(2x-1) \\ &= (3x^2+3x-6)(2x-1) \\ &= 6x^3-3x^2+6x^2-3x-12x+6 \\ &= 6x^3+3x^2-15x+6 \\ &\quad \begin{matrix} a & b & c & d \end{matrix} \end{aligned}$$

Q4 $f(x) = (x-3)(x+1)(x+2)$
 $= (x^2 + x - 3x - 3)(x+2)$
 $= (x^2 - 2x - 3)(x+2)$
 $= x^3 + 2x^2 - 2x^2 - 4x - 3x - 6$
 $= x^3 - 7x - 6$

Note
 No integer roots.
 $f(x) = x^3 + ax^2 + bx + c$
 $\Rightarrow a = 0 \quad b = -7 \quad c = -6$

Q5 Green $f(x) = x^3 + 2$.

Red $h(x) = 2x^3$

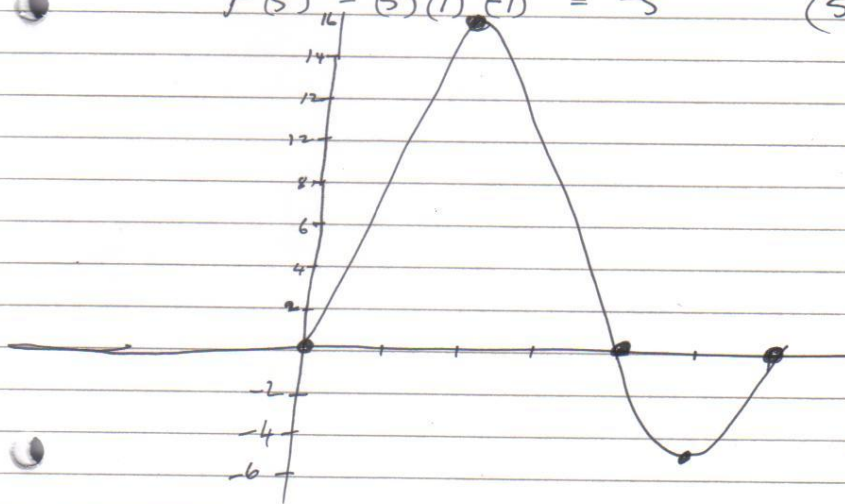
Blue $g(x) = x^3$

Q6

$f(x) = (x)(x-4)(x-6)$
 $x=0 \quad x=4 \quad x=6$

$f(2) = (2)(-2)(-4) = 16 \quad (2, 16)$

$f(5) = (5)(1)(1) = 5 \quad (5, 5)$



08 Roots: $x = -1$ $x = 1$ $x = 2$ $x = 2$
factors: $(x+1)(x-1)(x-2)(x-2)$

$$(x^2-1)(x^2-4x+4)$$
$$x^4 - 4x^3 + 4x^2 - x^2 + 4x - 4$$
$$x^4 - 4x^3 + 3x^2 + 4x - 4$$

Test (0,4)

$$4 = a[(0)^4 - 4(0)^3 + 3(0)^2 + 4(0) - 4]$$

$$4 = a(-4)$$

$$-1 = a$$

\Rightarrow function is $f(x) = -1(x^4 - 4x^3 + 3x^2 + 4x - 4)$
 $= -x^4 + 4x^3 - 3x^2 - 4x + 4$
 $ax^4 + bx^3 + cx^2 + dx + e$

$$a = -1 \quad b = 4 \quad c = -3 \quad d = -4 \quad e = 4$$

Q9 Looking at the graphs $f(x)$ is x^3
~~one~~ $g(x)$ is $-x^3$

$f(x)$ cuts y axis at 4

$g(x)$ is under x axis and cuts y at -2.

$$\Rightarrow a = -2.$$

$f(x)$ is opposite sign and is double the number of units on the y axis.

$$\begin{aligned} f(x) &= (x+2)(x+2)(x-1)(x-1) \\ &= (x^2+4x+4)(x^2-2x+1) \\ &= x^4 - 2x^3 + x^2 + 4x^3 - 8x^2 + 4x + 4x^2 - 8x + 4 \\ &= x^4 + 2x^3 - 3x^2 - 4x + 4 \end{aligned}$$

Test (0,4)

$$4 = (0)^4 + 2(0)^3 - 3(0)^2 - 4(0) + 4$$
$$4 = 4 \quad \text{True.}$$

$g(x)$ is $\frac{1}{2} f(x)$ and change the sign

$$\Rightarrow g(x) = \frac{1}{2}x^4 + x^3 - \frac{3}{2}x^2 - 2x + 2$$

Q11

$$x = -\frac{1}{2}, x = 3, x = 6$$

$$(2x+1)(x-3)(x-6)$$

$$(2x^2 - 5x - 3)(x-6)$$

$$2x^3 - 12x^2 - 5x^2 + 30x - 3x + 18$$

$$f(x) = 2x^3 - 17x^2 + 27x + 18$$

Test (1, 30)

$$30 = a(2 - 17 + 27 + 18)$$

$$30 = a(30)$$

$$1 = a$$

$$\Rightarrow f(x) = 2x^3 - 17x^2 + 27x + 18$$

Q12

$$f(x) = -3x^3 + 17x^2 + bx - 8$$

$$x = 2 \Rightarrow -3(2)^3 + 17(2)^2 + b(2) - 8 = 0$$

$$-24 + 68 + 2b - 8 = 0$$

$$2b = -36$$

$$b = -18$$

$$x = 2 \quad x = 4 \quad \Rightarrow (x-2)(x-4)$$

$$x^2 - 6x + 8$$

$$\begin{array}{r} x^2 - 6x + 8 \quad \left| \begin{array}{l} -3x - 1 \\ -3x^3 + 17x^2 - 18x - 8 \\ + 3x^3 + 18x^2 + 24x \end{array} \right. \end{array}$$

$$-x^2 + 6x - 8$$

$$+x^2 - 6x + 8$$

0

$$-3x - 1 = 0$$

$$-3x = 1$$

$$x = -\frac{1}{3} \text{ final Root.}$$