Solutions to ex 2.1

Text and Tests 2

Algebra 2

Ex 2.1

Q1 a (ii) $x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

$x = 3 \quad x = 4$

b (ii) $2x^2 + 7x - 15 = 0$

$(2x - 3)(x + 5) = 0$

$2x = 3$

$x = 3/2 \quad x = -5$

c (ii) $9x^2 + 3x - 20 = 0$

$(3x + 5)(3x - 4) = 0$

$3x = -5 \quad 3x = 4$

$x = -5/3 \quad x = 4/3$

d (ii) $3x^2 - 10x = 0$

$x(3x - 10) = 0$

$x = 0 \qquad 3x = 10$

$x = 10/3$

e (ii) $10 + x - 3x^2 = 0$

$3x^2 - x - 10 = 0$

$(3x + 5)(x - 2) = 0$

$3x = -5$

$x = -5/3 \qquad x = 2$

f (ii) $(x - 3)(4x^2 - 4) = 0$

$x = 3 \qquad 4x^2 = 4$

$x^2 = 1$

$x = \pm 1$

g (ii) $(2x + 8)(x^2 - 2x - 15) = 0$

$(2x + 8)(x - 5)(x + 3) = 0$

$2x = -8 \qquad x = 5 \qquad x = -3$

$x = -4$

Q2 a (ii) $x^2 + 3x - 2 = 0 \qquad a = 1 \quad b = 3 \quad c = -2$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 8}}{2}$$

$$x = \frac{-3 \pm \sqrt{17}}{2}$$

$x = 0.6 \qquad x = -3.6$

b(ii) $3x^2 - 8x + 1 = 0$ $a = 3$ $b = -8$ $c = 1$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(3)(1)}}{2(3)}$$

$$x = \frac{8 \pm \sqrt{64 - 12}}{6}$$

$$x = \frac{8 \pm \sqrt{52}}{6}$$

$x = 2.5$ $\quad$ 0.1

Q3 a (ii) $2x^2 - 12x - 5 = 0$ $a = 2$ $b = -12$ $c = -5$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(2)(-5)}}{2(2)}$$

$$x = \frac{12 \pm \sqrt{144 + 40}}{4}$$

$$x = \frac{12 \pm \sqrt{184}}{4}$$

$$x = \frac{6 \pm \sqrt{46}}{2}$$

b (ii) $5x^2+4x-2=0$ $a=5$ $b=4$ $c=-2$

$$x = \frac{-4 \pm \sqrt{(4)^2-4(5)(-2)}}{2(5)}$$

$$x = \frac{-4 \pm \sqrt{16+40}}{10}$$

$$x = \frac{-4 \pm \sqrt{56}}{10} = \frac{-2 \pm \sqrt{14}}{5}$$

Q4 a (ii) $\frac{1}{x-1} + \frac{4}{x} = \frac{3}{1}$

$$x + 4(x-1) = 3(x)(x-1)$$
$$x + 4x - 4 = 3x^2 - 3x$$
$$0 = 3x^2 - 8x + 4$$
$$0 = (3x-2)(x-2)$$
$$3x = 2$$
$$x = 2/3 \qquad x = 2.$$

b (ii) $\frac{x+2}{x-4} = \frac{2x+1}{x-2}$

$$(x+2)(x-2) = (2x+1)(x-4)$$
$$x^2 - 4 = 2x^2 - 7x - 4$$
$$0 = x^2 - 7x$$
$$0 = x(x-7)$$
$$x = 0 \qquad x = 7$$

(iii) back a page.

Q4

b (iii) $\frac{2}{x-2} + \frac{3}{x} = \frac{5}{x-4}$

$2(x)(x-4) + 3(x-2)(x-4) = 5(x)(x-2)$

$(3x-6)$

$2x^2 - 8x + 3x^2 - 18x + 24 = 5x^2 - 10x$

$+16x = +24$

$2x = 3$

$x = 3/2$

Q5

(i) $x^4 - 7x^2 + 10 = 0$ let $u = x^2 \Rightarrow u^2 = x^4$

$u^2 - 7u + 10 = 0$

$(u - 5)(u - 2) = 0$

$u = 5 \quad u = 2$

$\Rightarrow x^2 = 5 \quad x^2 = 2$

$x = \pm\sqrt{5} \quad x = \pm\sqrt{2}$

check:

$(\sqrt{5})^4 - 7(\sqrt{5})^2 + 10 = 0$

$25 - 35 + 10 = 0$ ✓

$(\sqrt{2})^4 - 7(\sqrt{2})^2 + 10 = 0$

$4 - 14 + 10 = 0$ ✓

(iv) $2(h-2)^2 - 3(h-2) - 4 = 0 \quad x = h - 2$

$2x^2 - 3x - 4 = 0 \quad a = 2 \quad b = -3 \quad c = -4$

$(2x \quad)(x \quad) = 0$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)}$

$x = \frac{3 \pm \sqrt{9 + 32}}{4}$

$x = \frac{3 \pm \sqrt{41}}{4}$

$\Rightarrow h - 2 = \frac{3 \pm \sqrt{41}}{4}$

$h = \frac{3 \pm \sqrt{41}}{4} + \frac{2}{1}$

$h = \frac{3 \pm \sqrt{41} + 8}{4} = \frac{11 \pm \sqrt{41}}{4}$

(b) (ii) $(2y-3)^2 - 1 = 0$ let $x = 2y-3$

$x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$x = -1 \quad x = 1$

$\Rightarrow \quad 2y-3 = -1 \quad 2y-3 = 1$

$2y = 2 \quad 2y = 4$

$y = 1 \quad y = 2.$

(c) $\left(y + \frac{4}{y}\right)^2 - 9\left(y + \frac{4}{y}\right) + 20 = 0$ $\quad x = y + \frac{4}{y}$

$x^2 - 9x + 20 = 0$

$(x-5)(x-4) = 0$

$x = 5 \quad x = 4$

$\Rightarrow \quad y + \frac{4}{y} = 5 \quad y + 4/y = 4$

$y^2 + 4 = 5y \quad y^2 + 4 = 4y$

$y^2 - 5y + 4 = 0 \quad y^2 - 4y + 4 = 0$

$(y-4)(y-1) = 0 \quad (y-2)(y-2) = 0$

$y = 4 \quad y = 1 \quad y = 2 \quad y = 2.$

check $\left(4 + \frac{4}{4}\right)^2 - 9\left(4 + \frac{4}{4}\right) + 20 = 0$

$25 - 45 + 20 = 0$ ✓

$\left(1 + \frac{4}{1}\right)^2 - 9\left(1 + \frac{4}{1}\right) + 20 = 0$

$25 - 45 + 20 = 0$ ✓

$\left(2 + \frac{4}{2}\right)^2 - 9\left(2 + 4/2\right) + 20 = 0$

$16 - 36 + 20 = 0$ ✓

Q6 $2x^2 - \sqrt{3}x - 3 = 0$ $a = 2$ $b = -\sqrt{3}$ $c = -3$

$$x = \frac{\sqrt{3} \pm \sqrt{(-\sqrt{3})^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{\sqrt{3} \pm \sqrt{3 + 24}}{4}$$

$$x = \frac{\sqrt{3} \pm \sqrt{27}}{4}$$

$x = \sqrt{3}$

$x = \frac{-\sqrt{3}}{2}$

Q7

(a) $x = -4.2$ $x = 1.2$

(b) $x = -2.3$ $x = 1.3$

(c) $x = -9.6$ $x = -0.5$

(d) $-1.7 < x < 0.6$

(e) $x = -1.7$ $x = 0.7$

(f) $x = -3$ $x = 1$

(g) $x = -3.5$ $x = 0.6$

(h) $-4.1 < x < -0.8$

Q8 Does not cut x axis

Q9 $x_1 = -2.5$ $x_2 = -22.5$

$(x_2 - x_1) = (-22.5 - -2.5) = -20$ $-23 + 2 = -21$

$(x_2 + x_1) = (-22.5 + -2.5) = -25$ $23 - 2 = 25$

Q10 (a) $x = -0.5$ $x = 2$

(b) $x = 0.8$

(c) $x = -0.6$ $x = 2.4$

Q11 $2x + 3\sqrt{x} = 5$ $u = \sqrt{x}$ $u^2 = x$

$2u^2 + 3u - 5 = 0$

$(2u + 5)(u - 1)$

$2u = -5$ $u = 1$

$u = -5/2$

$\sqrt{x} = -5/2$ $\sqrt{x} = 1$

$x = 25/4$ $x = 1$

Test $2\left(\frac{25}{4}\right) + 3\sqrt{\left(\frac{25}{4}\right)} = 5$ $2(1) + 3(\sqrt{1}) = 5$

$12.5 + \frac{3(5/4)}{3.75} \neq 5.$ $2 + 3 = 5$ ✓

(ii) $x - 3\sqrt{x} - 4 = 0$ $u = \sqrt{x} \Rightarrow u^2 = x.$

$u^2 - 3u - 4 = 0$

$(u - 4)(u + 1) = 0$

$u = 4$ $u = -1$

$\sqrt{x} = 4$ $\sqrt{x} = -1$

$x = 16$ $x = 1$

$16 - 3\sqrt{16} - 4$ $1 - 3(1) - 4 = 0$

$16 - 12 - 4 = 0$ ✓ $1 - 3 - 4 \neq 0$ ✗

$x = 16.$

Q12 (i) $x^2 - \sqrt{7}x - 14 = 0$.

$$x = \frac{\sqrt{7} \pm \sqrt{(-\sqrt{7})^2 - 4(1)(-14)}}{2(1)}$$

$$x = \frac{\sqrt{7} \pm \sqrt{7 + 56}}{2}$$

$$= \frac{\sqrt{7} \pm \sqrt{63}}{2}$$

$$= \frac{\sqrt{7} \pm 3\sqrt{7}}{2}$$

$$\frac{\sqrt{7} + 3\sqrt{7}}{2} \qquad \frac{\sqrt{7} - 3\sqrt{7}}{2}$$

$$= \frac{4\sqrt{7}}{2} \qquad = \frac{-2\sqrt{7}}{2}$$

$$= 2\sqrt{7} \qquad = -\sqrt{7}$$

(ii)

$$2x^2 + 7\sqrt{5}x + 15 = 0$$

$$x = \frac{-7\sqrt{5} \pm \sqrt{(7\sqrt{5})^2 - 4(2)(15)}}{2(2)}$$

$$= \frac{-7\sqrt{5} \pm 5\sqrt{5}}{4}$$

$$\frac{-7\sqrt{5} + 5\sqrt{5}}{4} \qquad \frac{-7\sqrt{5} - 5\sqrt{5}}{4}$$

$$= \frac{-2\sqrt{5}}{4} \qquad = \frac{-12\sqrt{5}}{4}$$

$$= -\frac{\sqrt{5}}{2} \qquad = -3\sqrt{5}$$