

• Ex 2.9

Q1 $(x-3) \Rightarrow x=3$
 $(3)^2 - 8(3) + 15 = 0$
 $9 - 24 + 15 = 0$ True $\Rightarrow x-3$ is a factor

Q2 $(x-1) \Rightarrow x=1$
 $(1)^3 - (1)^2 - 9(1) + 9 = 0$
 $1 - 1 - 9 + 9 = 0$ True $\Rightarrow x-1$ is a factor

Q3 $(x+2) \Rightarrow x=-2$
 $(-2)^3 + 6(-2)^2 + 11(-2) + 6 = 0$
 $-8 + 24 - 22 + 6 = 0$ True $\Rightarrow x+2$ a factor

Q4 $(x-2) \Rightarrow x=2$
 $2(2)^3 - 3(2)^2 - 12(2) + 20 = 0$
 $16 - 12 - 24 + 20 = 0$ True $\Rightarrow x-2$ a factor

Q5 $(x-2) \Rightarrow x=2$
 $(2)^3 - 5(2)^2 + 8(2) - 4 = 0$
 $8 - 20 + 16 - 4 = 0$ True $\Rightarrow x-2$ a factor

Q6 $(2x-1) \Rightarrow 2x=1 \Rightarrow x=\frac{1}{2}$
 $2(\frac{1}{2})^3 + 7(\frac{1}{2})^2 + 2(\frac{1}{2}) - 3 = 0$
 $\frac{1}{4} + \frac{7}{4} + 1 - 3 = 2 + 1 - 3 = 0$ True $2x-1$ a factor

Q7 $(2x+1) \Rightarrow 2x=-1 \Rightarrow x=-\frac{1}{2}$
 $2(-\frac{1}{2})^3 - (-\frac{1}{2})^2 - 5(-\frac{1}{2}) - 2 = 0$
 $-\frac{1}{4} - \frac{1}{4} + \frac{5}{2} - 2 = 0$
 $2 - 2 = 0$ True $\Rightarrow 2x+1$ a factor

Q8 $(x-1) \Rightarrow x=1$
 $(1)^3 + k(1)^2 - 8 = 0$
 $1 + k - 8 = 0$
 $k = 8$

Q10 $(x-3) \Rightarrow x=3$
 $(3)^3 - 2(3)^2 - 5(3) + 6 = 0$
 $27 - 18 - 15 + 6 = 0$ True $\Rightarrow x-3$ a factor

$$\begin{array}{r} x^2 + x - 2 \\ x-3 \overline{) x^3 - 2x^2 - 5x + 6} \\ \underline{-x^3 + 3x^2} \\ x^2 - 5x \\ \underline{-x^2 + 3x} \\ -2x + 6 \\ \underline{+2x - 6} \\ 0 \end{array}$$

$x^2 + x - 2$
 $(x+2)(x-1)$ factors are $(x-3)(x+2)(x-1)$

~~Q10 $(x-3) \Rightarrow x=3$
 $(3)^3 - 2(3)^2 - 5(3) + 6 = 0$
 $27 - 18 - 15 + 6 = 0$~~

Q9 $(x+2) \Rightarrow x=-2$
 $(-2)^3 + 6(-2)^2 + p(-2) + 6 = 0$
 $-8 + 24 - 2p + 6 = 0$
 $-2p = -22$
 $p = 11$

Q12 (ii) $x^3 - 8x^2 + 19x - 12$
 $f(1) = (1)^3 - 8(1)^2 + 19(1) - 12$
 $= 1 - 8 + 19 - 12 = 0 \Rightarrow (x-1) \text{ a factor}$

$$\begin{array}{r} x^2 - 7x + 12 \\ x-1 \overline{) x^3 - 8x^2 + 19x - 12} \\ \underline{-x^3 + x^2} \\ -7x^2 + 19x \\ \underline{+7x^2 + 7x} \\ 12x - 12 \\ \underline{-12x + 12} \\ 0 \end{array}$$

$x^2 - 7x + 12$
 $(x-3)(x-4)$ factors are $(x-1)(x-3)(x-4)$

Q12 (iv) $3x^3 - 4x^2 - 3x + 4$
 $f(1) = 3(1)^3 - 4(1)^2 - 3(1) + 4$
 $= 3 - 4 - 3 + 4 = 0 \Rightarrow (x-1) \text{ a factor}$

$$\begin{array}{r} 3x^2 - x - 4 \\ x-1 \overline{) 3x^3 - 4x^2 - 3x + 4} \\ \underline{-3x^3 + 3x^2} \\ -x^2 - 3x \\ \underline{+x^2 + x} \\ -4x + 4 \\ \underline{+4x + 4} \\ 0 \end{array}$$

$3x^2 - x - 4$
 $(3x-4)(x+1)$ factors are $(3x-4)(x+1)(x-1)$

Q12 (vi)

$$2x^3 - 3x^2 - 12x + 20$$

$$f(1) = 2(1)^3 - 3(1)^2 - 12(1) + 20$$

$$2 - 3 - 12 + 20 \neq 0$$

$$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 20$$

$$16 - 12 - 24 + 20 = 0 \Rightarrow (x-2) \text{ a factor}$$

$$\begin{array}{r} 2x^2 + x - 10 \\ x-2 \overline{) 2x^3 - 3x^2 - 12x + 20} \\ \underline{-2x^3 + 4x^2} \\ x^2 - 12x \\ \underline{-x^2 + 2x} \\ -10x + 20 \\ \underline{+10x - 20} \\ 0 \end{array}$$

$$2x^2 + x - 10$$

$$(2x+5)(x-2) \quad \text{factors: } (x-2)(2x+5)(x-2)$$

Q14

$$(x+2) \Rightarrow x = -2$$

$$(-2)^3 + a(-2)^2 - (-2) - 2 = 0$$

$$-8 + 4a + 2 - 2 = 0$$

$$4a = 8$$

$$a = 2$$

$$\begin{array}{r} x^2 - 1 \\ x+2 \overline{) x^3 + 2x^2 - x - 2} \\ \underline{-x^3 + 2x^2} \\ -x - 2 \\ \underline{+x + 2} \\ 0 \end{array}$$

$$(x^2 - 1)$$

$$(x+1)(x-1) \quad \text{factors are: } (x+2)(x+1)(x-1)$$

Q17

(iii)

$$3x^3 - 4x^2 - 3x + 4$$

$$f(1) = 3(1)^3 - 4(1)^2 - 3(1) + 4$$

$$3 - 4 - 3 + 4 = 0 \Rightarrow (x-1) \text{ a factor}$$

$$\begin{array}{r} 3x^2 - x - 4 \\ x-1 \overline{) 3x^3 - 4x^2 - 3x + 4} \\ \underline{-3x^3 + 3x^2} \\ -x^2 - 3x \\ \underline{+x^2 + x} \\ -4x + 4 \\ \underline{+4x - 4} \\ 0 \end{array}$$

$$3x^2 - x - 4 \\ (3x - 4)(x + 1)$$

factors are $(x-1)$ $(3x-4)$ $(x+1)$

\Rightarrow Solutions are $x=1$ $3x=4$ $x=-1$
 $x=4/3$

Q.17 (iv) $x^3 - 7x - 6$
 $f(-1) = (-1)^3 - 7(-1) - 6$
 $-1 + 7 - 6 = 0 \Rightarrow (x+1) \text{ a factor}$

$$\begin{array}{r} x^2 - x - 6 \\ x+1 \overline{) x^3 + 0x^2 - 7x - 6} \\ \underline{-x^3 + x^2} \\ -x^2 - 7x \\ \underline{+x^2 + x} \\ -6x - 6 \\ \underline{+6x + 6} \\ 0 \end{array}$$

$$x^2 - x - 6 \\ (x-3)(x+2)$$

factors are $(x+1)(x-3)(x+2)$

\Rightarrow Solutions are $x = -1, x = 3, x = -2$

Q18 $x+1 \Rightarrow x = -1$

$$2(-1)^3 + a(-1)^2 + b(-1) - 3 = 0$$

$$-2 + a - b - 3 = 0$$

$$\boxed{a - b = 5}$$

$x+3 \Rightarrow x = -3$

$$2(-3)^3 + a(-3)^2 + b(-3) - 3 = 0$$

$$-54 + 9a - 3b - 3 = 0$$

$$9a - 3b = 57 \quad (\div 3)$$

$$\boxed{3a - b = 19}$$

~~$$-a + b = -5$$~~

$$3a - b = 19$$

$$2a = 14$$

$$a = 7$$

$$a - b = 5$$

$$7 - b = 5$$

$$-b = -2$$

$$b = 2$$

$$2x^3 + 7x^2 + 2x - 3$$

find the 3rd factor:

$$\begin{array}{r} 2x^2 + 5x - 3 \\ x+1 \overline{) 2x^3 + 7x^2 + 2x - 3} \\ \underline{-2x^3 + 2x^2} \\ 5x^2 + 2x \\ \underline{-5x^2 + 5x} \\ -3x - 3 \\ \underline{+3x + 3} \\ 0 \end{array}$$

$$2x^2 + 5x - 3$$

$$(2x - 1)(x + 3)$$

\therefore 3rd factor is $2x - 1$.

factors are $(x+1)(x+3)(2x-1)$

\Rightarrow Solutions are $x = -1$ $x = -3$ $x = 1/2$.

Q21 Divisible by $x^2 - 2x - 3$
 $\Rightarrow (x-3)(x+1)$ are factors
 $\Rightarrow x=3$ and $x=-1$

$$x=3 \Rightarrow a(3)^3 + 8(3)^2 + b(3) + 6 = 0$$

$$27a + 72 + 3b + 6 = 0$$

$$27a + 3b = -78 \quad (\div 3)$$

$$\boxed{9a + b = -26}$$

$$x=-1 \Rightarrow a(-1)^3 + 8(-1)^2 + b(-1) + 6 = 0$$

$$-a + 8 - b + 6 = 0$$

$$\boxed{-a - b = -14}$$

$$\begin{array}{r} 9a + b = -26 \\ -a - b = -14 \\ \hline 8a = -40 \\ a = -5 \end{array} \qquad \begin{array}{r} -a - b = -14 \\ 5 - b = -14 \\ -b = -19 \\ b = 19 \end{array}$$

$$\Rightarrow f(x) = -5x^3 + 8x^2 + 19x + 6$$

$$\begin{array}{r} -5x^2 + 13x + 6 \\ x+1 \overline{) -5x^3 + 8x^2 + 19x + 6} \\ \underline{+5x^3 + 5x^2} \\ 13x^2 + 19x \\ \underline{-13x^2 + 13x} \\ 6x + 6 \\ \underline{-6x + 6} \\ 0 \end{array}$$

$$-5x^2 + 13x + 6$$

$$(-5x-2)(x-3)$$

\Rightarrow factors are: $(x+1)(x-3)(-5x-2)$
 \Rightarrow Solutions are $x=-1$ $x=3$ $5x=-2$
 $x=-2/5$

Q22 (i) $ax^3 - b = c$

$$ax^3 = c + b$$

$$x^3 = \frac{c+b}{a}$$

$$x = \sqrt[3]{\frac{c+b}{a}} \quad \text{or} \quad \left(\frac{c+b}{a}\right)^{\frac{1}{3}}$$

(ii) $a(x+b)^3 = c$

$$(x+b)^3 = c/a$$

$$x+b = \sqrt[3]{c/a}$$

$$x = \sqrt[3]{c/a} - b \quad \text{or} \quad \left(\frac{c}{a}\right)^{\frac{1}{3}} - b$$