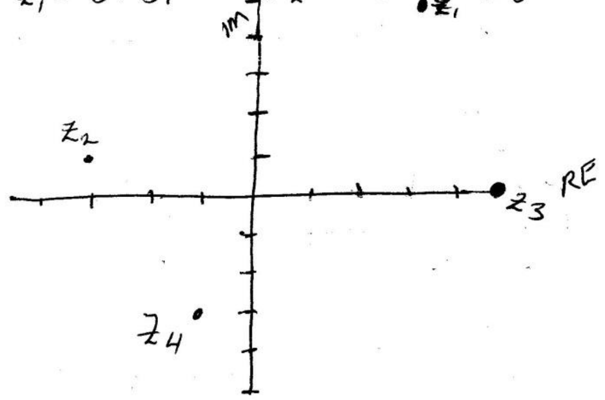
Ex 3·4

Q1 $z_1 = 3+5i$ $\quad z_2 = -3+i$ $\quad z_3 = 5i$ $\quad z_4 = -1-3i$



Q2 (i) $z_1 = 2+i$ $\qquad$ (ii) $z_2 = -4+3i$

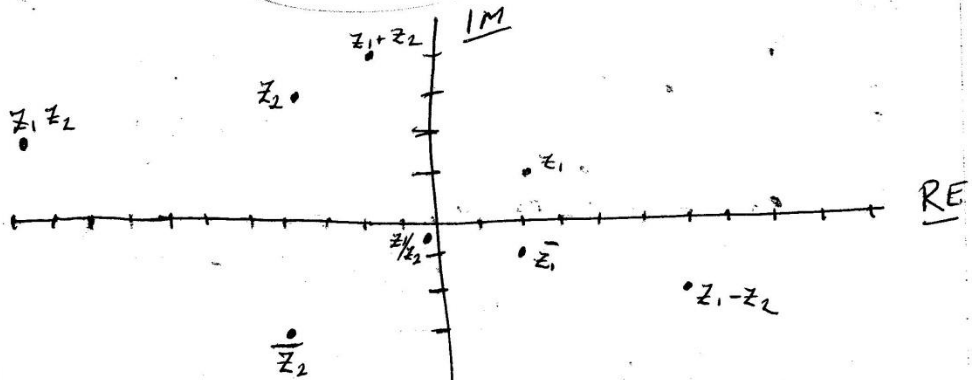
(iii) $\bar{z}_1 = 2-i$ $\qquad$ (iv) $\bar{z}_2 = -4-3i$

(v) $z_1+z_2 = -2+4i$ $\qquad$ (vi) $z_1-z_2 = 6-2i$

(vii) $z_1 z_2 = -8+6i-4i+3i^2 = -11+2i$

(viii) $\dfrac{z_1}{z_2} = \dfrac{2+i}{-4+3i} \times \dfrac{-4-3i}{-4-3i} = \dfrac{-8-6i-4i-3i^2}{16+9} = \dfrac{-5-10i}{25}$

$= -\dfrac{1}{5} - \dfrac{2}{5}i$



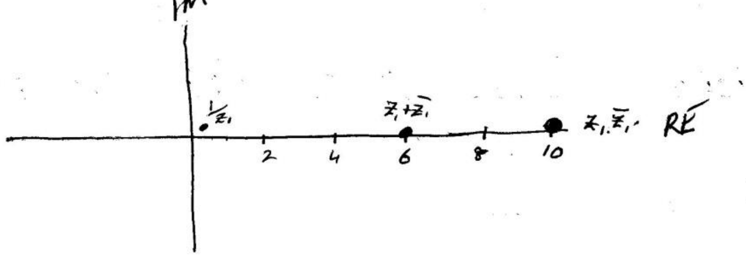
Q3 $z_1 = 3-i \quad z_2 = 2+4i$

(i) $z_1 . \bar{z}_1 = (3-i)(3+i) = 9+1 = 10$

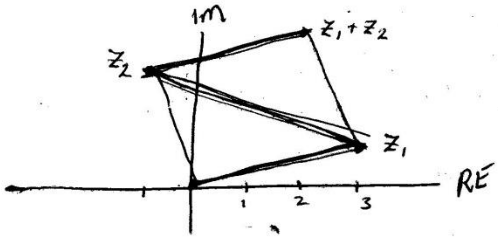
(ii) $z_1 + \bar{z}_1 = 3-i + 3+i = 6+0i$

(iii) $\frac{1}{z_1} = \frac{1}{3-i} \times \frac{3+i}{3+i} = \frac{3+i}{10} = \frac{3}{10} + \frac{1}{10}i$

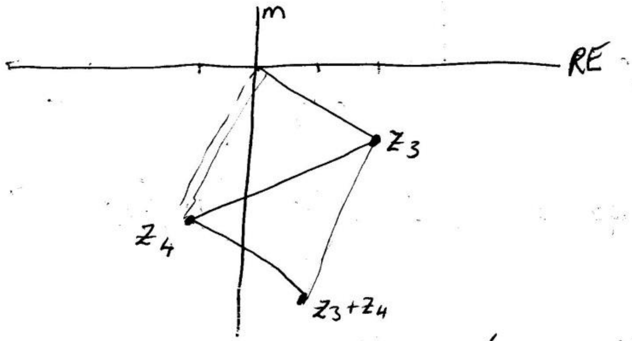
(iv) $z_1 z_2 = (3-i)(2+4i) = 6+12i-2i+4i^2 = 10+10i$



Q4 (a) $z_1 = 3+i \quad z_2 = -1+3i \quad z_1+z_2 = 2+4i$



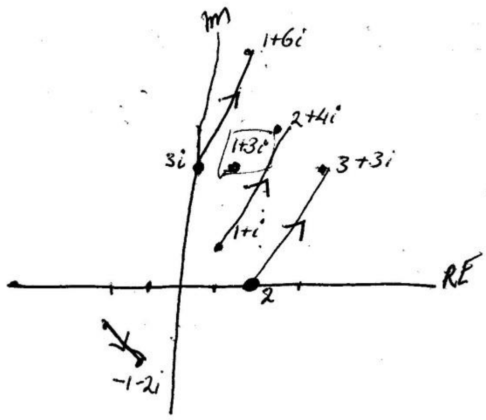
(b) $z_3 = 2-2i \quad z_4 = -1-4i \quad z_3+z_4 = 1-6i$



(c) $0, z_1, z_2, z_1+z_2$ join to make a //gram.

Q5 $z = 1 + 3i$

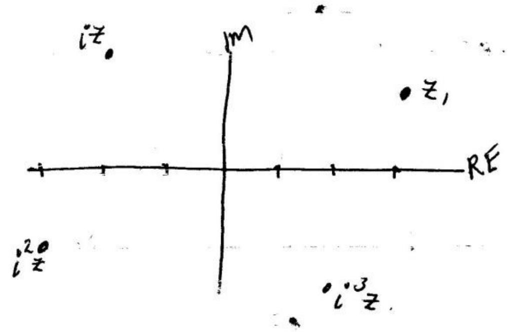
(i) 2
(ii) $2 + z = 3 + 3i$
(iii) $3i$
(iv) $3i + z = 1 + 6i$
(v) $1 + i$
(vi) $1 + i + z = 2 + 4i$
(vii) $3i + z = 1 + 6i$
(viii) $-2 - i + z = -1 - 2i$



Adding a complex N°, Translates the point.

Q6 $z = 3 + 2i$

(i) $iz = i(3+2i) = 3i + 2i^2 = \underline{-2+3i}$
(ii) $i^2 z = -1(3+2i) = \underline{-3-2i}$
(iii) $i^3 z = -i(3+2i) = -3i - 2i^2 = \underline{2-3i}$



Mult by i, i^2, i^3, etc Rotates the point

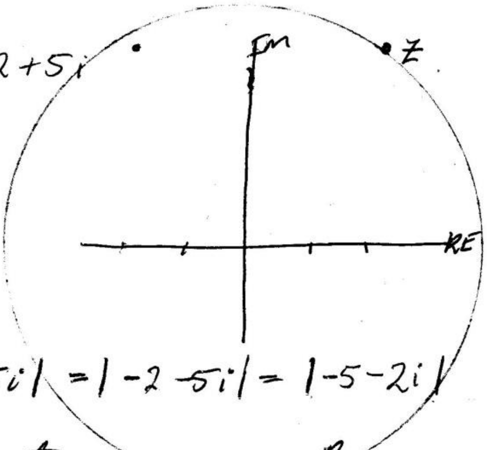
Q7 (i) $|5+2i| = \sqrt{5^2+2^2} = \sqrt{29}$

(ii) $|4-2i| = \sqrt{4^2+(-2)^2} = \sqrt{20} = 2\sqrt{5}$

(iii) $|-2-4i| = \sqrt{(-2)^2+(-4)^2} = \sqrt{20} = 2\sqrt{5}$

(iv) $|-3+i| = \sqrt{(-3)^2+(1)^2} = \sqrt{10}$

Q8 $z = 2+5i$



$|Z| = |-2+5i| = |-2-5i| = |-5-2i|$

All Points on circumference of circle

Q9

(i) $\dfrac{3+i}{-2-3i} \times \dfrac{-2+3i}{-2+3i} = \dfrac{-6+9i-2i\overset{\ominus}{+}3\cancel{i^2}}{4+9} = \dfrac{-9+7i}{13}$

$= \dfrac{-9}{13} + \frac{7}{13}i$

$\Rightarrow \left|\dfrac{3+i}{-2-3i}\right| = \sqrt{\left(\frac{-9}{13}\right)^2 + \left(\frac{-7}{13}\right)^2} = \dfrac{\sqrt{130}}{13}$

(ii) $(4+2i)(3-i) = 12-4i+6i\overset{\oplus}{-}2\cancel{i^2} = 14+2i$

$\Rightarrow |(4+2i)(3-i)| = |14+2i| = \sqrt{14^2+(2)^2} = 10\sqrt{2}$

(iii) $\frac{1}{3+5i} \times \frac{3-5i}{3-5i} = \frac{3-5i}{9+25} = \frac{3}{34} - \frac{5}{34}i$

$\Rightarrow \left|\frac{1}{3+5i}\right| = \left|\frac{3}{34} - \frac{5}{34}i\right| = \sqrt{(3/34)^2 + (-5/34)^2} = \frac{\sqrt{34}}{34}$

Q10 $z_1 = -2-3i \qquad z_2 = 3+i$

$\frac{z_1}{z_2} = \frac{-2-3i}{3+i} \times \frac{3-i}{3-i} = \frac{-6+2i-9i \cancel{+3i^2}\,-3}{9+1}$

$= \frac{-9-7i}{10} = \frac{-9}{10} - \frac{7}{10}i$

$\frac{|z_1|}{|z_2|} = \left|\frac{z_1}{z_2}\right|$

$|z_1| = \sqrt{(-2)^2+(-3)^2} = \sqrt{13}$
$|z_2| = \sqrt{(3)^2+(1)^2} = \sqrt{10}$

$\therefore \quad \frac{|z_1|}{|z_2|} = \frac{\sqrt{13}}{\sqrt{10}} = \frac{\sqrt{130}}{10}$

$\left|\frac{z_1}{z_2}\right| = \left|\frac{-9}{10} - \frac{7}{10}i\right| = \sqrt{\left(\frac{-9}{10}\right)^2 + \left(\frac{-7}{10}\right)^2} = \frac{\sqrt{130}}{10}$

$\Rightarrow$ TRUE $\quad \frac{|z_1|}{|z_2|} = \left|\frac{z_1}{z_2}\right|$

Q11 $\dfrac{1}{u} = \dfrac{1}{v} + \dfrac{1}{w}$ $(\times uvw)$ $\quad v = 3+4i$
$\quad w = 4-3i$ find u.

$$vw = uw + uv$$
$$vw = u(w+v)$$
$$\frac{vw}{w+v} = u$$

$$\frac{(3+4i)(4-3i)}{(4-3i)+(3+4i)} = \frac{12 - 9i + 16i \overset{+}{-} 12i^2}{7+i} = \frac{24+7i}{7+i}$$

$$\frac{24+7i}{7+i} \times \frac{7-i}{7-i} = \frac{168 - 24i + 49i \overset{+}{-} 7i^2}{49+1} = \frac{175+25i}{50}$$

$$= \frac{7}{2} + \frac{1}{2}i = u.$$

Q12 $z = 4 - 2i$

$$|z| = \sqrt{4^2+(-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$
$$|2z| = |8-4i| = \sqrt{64+16} = 4\sqrt{5}$$
$$|3z| = |12-6i| = \sqrt{144+36} = 6\sqrt{5}$$

$2|z| = |2z|$?
$2(2\sqrt{5}) = 4\sqrt{5}$
$4\sqrt{5} = 4\sqrt{5}$ True

Q13 $|z| = |\bar{z}|$ let $z = a+bi$

$$|a+bi| = |a-bi|$$
$$\sqrt{a^2+b^2} = \sqrt{a^2+(-b)^2}$$
$$\sqrt{a^2+b^2} = \sqrt{a^2+b^2} \quad \therefore \text{ True.}$$

Q14 $z_1 = s+8i \qquad z_2 = t+8i$

(i) $|z_1| = 10$

$$\sqrt{s^2+64} = 10$$
$$s^2+64 = 100$$
$$s^2 = 36$$
$$s = \pm 6$$

(ii) $|z_2| = 2|z_1|$
$$|t+8i| = 2|s+8i|$$
$$\sqrt{t^2+64} = 2\sqrt{s^2+64}$$

$$t^2+64 = 4(s^2+64)$$
$$t^2+64 = 4s^2+256$$
$$t^2 = 4s^2+192$$

at $s=6$ $\qquad t^2 = 144+192$
$$t^2 = 336$$
$$t = \pm 4\sqrt{21}$$

at $s=-6$ $\quad t^2 = 144+192$ same as above

$\Rightarrow \quad s = \pm 6 \qquad t = \pm 4\sqrt{21}$

Q15 $\frac{i}{1-i} \times \frac{1+i}{1+i} = \frac{1+i}{1^2+1^2} = \frac{1+i}{2} = \frac{1}{2}+\frac{1}{2}i$

$\left|\frac{i}{1-i}\right| = \left|\frac{1}{2}+\frac{1}{2}i\right| = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

Q16

$|z-1|\,|z-1| = 1$

$\left(\sqrt{z^2+1}\right)\left(\sqrt{z^2+1}\right) = 1$

$z^2+1 = 1$

$z = 0 \Rightarrow 0+0i$

$\Rightarrow$ a circle of centre $(0,0)$ and radius $=1$

Q17 z_1 and z_2 must both be real or both Imaginary

or $z_2 = az_1$ ie. z_2 and z_1 must be on the same line from the origin.