Ex 3.6

Q1 (i) $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$

(ii) $\frac{dV}{dr} = \frac{dV}{dt} \cdot \frac{dt}{dr}$

(iii) $\frac{dM}{dt} = \frac{dM}{ds} \cdot \frac{ds}{dt}$

Q2 (i) $\frac{dA}{dt} = 8 \qquad \frac{dA}{dr} = 4.$

$$\frac{dr}{dt} = \frac{dr}{dA} \cdot \frac{dA}{dt}$$

$$= \frac{1}{4} \cdot 8 = \frac{8}{4} = 2.$$

(ii) $\frac{dV}{dt} = 8 \qquad \frac{dr}{dt} = 2$

$$\frac{dV}{dr} = \frac{dV}{dt} \times \frac{dt}{dr}$$

$$= 8 \times \frac{1}{2} = 4$$

Q3 $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$= (10) \cdot (2) = 20$

Q4 $A = \pi r^2$ $\quad \frac{dr}{dt} = 1$

$\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$= 2\pi r \times 1$

at $r = 5$ $\quad 2\pi(5) = 10\pi$

Q6 Given $\frac{dx}{dt} = 5$

Find $\frac{dA}{dt} = \frac{dx}{dt} \cdot \frac{dA}{dx}$

Area $= x^2$

$\Rightarrow \frac{dA}{dx} = 2x$

$\therefore \frac{dA}{dt} = \frac{dx}{dt} \cdot \frac{dA}{dx} = (5)(2x) = 10x$

at $x = 10$ $\quad \Rightarrow 10(10) = 100 \text{ cm}^2/\text{sec}$

Q8 $V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = 6$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dt} = \frac{dV}{dt} \cdot \frac{dr}{dV}$

$= (6) \cdot \left(\frac{1}{4\pi r^2}\right) = \frac{3}{2\pi r^2}$

at $r = 3$ $\quad \frac{3}{2\pi(3)^2} = \frac{3}{18\pi} = \frac{1}{6\pi}$ cm/sec

Q10 $P = 40$ $\quad \frac{dx}{dt} = 0.5$

$P = 2x + 2y = 40$

$x + y = 20$

$y = 20 - x$

(i) Area $\Rightarrow$ $x(20-x)$

$A = 20x - x^2$ $\quad \rightarrow \frac{dA}{dx} = (20 - 2x)$

(ii) find $\frac{dA}{dt} = \frac{dx}{dt} \times \frac{dA}{dx}$

$\frac{dA}{dt} = (0.5)(20 - 2x)$

at $x = 3$ $\quad (0.5)(20 - 2(3))$

$(0.5)(14)$

$= 7 \text{ cm}^2/\text{sec}$

Q11 $y = x - \frac{x^2}{40}$ $\quad \frac{dx}{dt} = 10\sqrt{2}$

$\rightarrow \frac{dy}{dx} = 1 - \frac{2x}{40}$

$1 - x/20$

$\frac{dy}{dt} = \frac{dx}{dt} \times \frac{dy}{dx}$

$\frac{dy}{dt} = (10\sqrt{2})(1 - \frac{x}{20})$

When $x = 10$ $\quad 10\sqrt{2}\left(1 - \frac{10}{20}\right)$

$10\sqrt{2}\left(\frac{1}{2}\right)$

$= \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

Q12 $\frac{dr}{dt} = 1\text{cm/sec}$ $r = 2\text{m} = 200\text{cm}$.

Find $\frac{dV}{dt}$.

(i) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$

$= (4\pi r^2)(1) = 4\pi r^2$

at $r = 200 \Rightarrow 4\pi(200)^2 = 160000\pi\ \text{cm}^3/\text{sec}$

(ii) Find $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$. $\frac{dV}{dr} = 4\pi r^2$

$\Rightarrow \frac{dr}{dV} = \frac{1}{4\pi r^2}$

$\frac{dr}{dt} = (160000\pi)\left(\frac{1}{4\pi r^2}\right)$

at $r = 5\text{m} = 500\text{cm}$

$= (160000\pi)\frac{1}{4\pi(500)^2} = \frac{4}{25} = 0.16\text{cm/sec}$

(iii) $SA = 4\pi r^2$

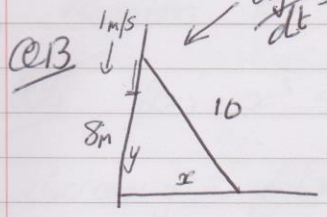
$\frac{dA}{dr} = 8\pi r$.

Find $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

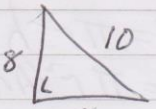
$\frac{dA}{dt} = (8\pi r)(0.16)$

at $r = 5\text{m} = 500\text{cm}$ $(8\pi\ 500)(0.16) = 640\pi\ \text{cm}^2/\text{sec}$

Q13



$\frac{dy}{dt} = 1$.



$10^2 = 8^2 + x^2$

$100 - 64 = x^2$

$\sqrt{36} = x^2$

$6 = x$

Find $\frac{dx}{dt}$

Ladder Top to ground at any time $= (8-y)m$.
Bottom of ladder to wall at any time $= (6+x)m$
Length of ladder is constant $= 10m$

Right Angled Triangle

$(8-y)^2 + (6+x)^2 = 10^2$ $\quad \left[\frac{dx}{dy}\right] \Rightarrow x =$ Terms of y.

$(6+x)^2 = 100 - (8-y)^2$ sq root both sides.

$6 + x = \sqrt{100 - (8-y)^2}$

$x = [100 - (8-y)^2]^{1/2} - 6$.

$\Rightarrow \frac{dx}{dy} = \frac{1}{2}\left(100 - (8-y)^2\right)^{-1/2} (-2)(8-y)(-1)$

$= \frac{16 - 2y}{2\sqrt{100 - (8-y)^2}} = \frac{8-y}{\sqrt{100-(8-y)^2}}$

$\frac{dx}{dt} = \frac{dx}{dy} \cdot \frac{dy}{dt}$

$= \frac{8-y}{\sqrt{100-(8-y)^2}} \cdot (1)$

Since $8 - y = 8 \Rightarrow y = 0$

$\Rightarrow \frac{dx}{dt} = \frac{8-0}{\sqrt{100-(8-0)^2}} = \frac{8}{\sqrt{100-64}} = \frac{8}{6} = 1\frac{1}{3}$ m/sec

Q14 $V = \pi r^2 h$

$V = \pi r^2 (4r)$

$V = 4r^3 \pi$

$\frac{dV}{dr} = 12r^2 \pi$

Given $\frac{dr}{dt} = 0.5$

Find $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$

$= (12r^2 \pi)(0.5)$

When $r = 6$ $\quad 12(6)^2 \pi (0.5) = 216\pi \text{ cm}^3/\text{sec}$

Q15 $\text{Cir} = 2\pi r$ $\quad \frac{dC}{dr} = 2\pi$ $\quad \text{Area} = \pi r^2$ $\quad \frac{dA}{dr} = 2r\pi$

$\frac{dC}{dA} = \frac{dC}{dr} \cdot \frac{dr}{dA}$

$= (2\pi)\left(\frac{1}{2r\pi}\right) = \frac{1}{r}$ $\quad$ QED.

Given $\frac{dA}{dt} = 2$.

Find $\frac{dC}{dt}$. $\quad \frac{dC}{dt} = \frac{dA}{dt} \times \frac{dC}{dA}$

$= (2)\left(\frac{1}{r}\right)$

at $r = 3$ $\quad 2\left(\frac{1}{3}\right) = 2/3 \text{ cm/sec}$.