Ex 3.8

Q1 $z_1 = 4\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ $\quad z_2 = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(i) $z_1 . z_2 = (4\times 2)\left[\cos\left(\frac{3\pi}{4}+\frac{\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}+\frac{\pi}{4}\right)\right]$

$= 8\left(\cos\pi + i\sin\pi\right)$

(ii) $\frac{z_1}{z_2} = \frac{4}{2}\left[\cos\left(\frac{3\pi}{4}-\frac{\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)\right]$

$= 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

Q2 $z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ find $z^2 = z . z$

$z^2 = (2\times 2)\left(\cos\left(\frac{\pi}{3}+\frac{\pi}{3}\right) + i\sin\left(\pi/3 + \pi/3\right)\right)$

$= 4\left(\cos\frac{2\pi}{3} + i\sin 2\pi/3\right)$

Q4 $4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \times 3\left(\cos\frac{\pi}{3} + i\sin\pi/3\right)$

$= 12\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

Q6 $2\left(\cos\pi/9 + i\sin\pi/9\right) . \frac{1}{3}\left(\cos\pi/9 + i\sin\pi/9\right) . 6\left(\cos\pi/9 + i\sin\pi/9\right)$

$= 2\times\frac{1}{3}\times 6\left(\cos\left(\pi/9+\pi/9+\pi/9\right) + i\sin\left(\pi/9+\pi/9+\pi/9\right)\right)$

$= 4\left(\cos\pi/3 + i\sin\pi/3\right)$

$= 4\left(\frac{1}{2} + i\,\sqrt{3}/2\right) = 2 + 2\sqrt{3}i$

Q7 $\left(\cos\frac{3\pi}{7} + i\sin\frac{3\pi}{7}\right)\left(\cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}\right)^2$ ← Note.

$= \left(\cos\frac{3\pi}{7} + i\sin\frac{3\pi}{7}\right)\left(\cos\frac{4\pi}{7} + i\sin\frac{4\pi}{7}\right)$

$= \cos\pi + i\sin\pi$

Q8 (a) $\left[2\left(\cos\pi/3 + i\sin\pi/3\right)\right]^3$

$= (2\times2\times2)\left(\cos\left(\frac{\pi}{3}+\pi/3+\pi/3\right) + i\sin\left(\pi/3+\pi/3+\pi/3\right)\right)$

$= 8\left(\cos\pi + i\sin\pi\right)$

(b) $\left[2\left(\cos\pi/3 + i\sin\pi/3\right)\right]^4$

$= 16\left(\cos 4\pi/3 + i\sin 4\pi/3\right)$

$2\times2\times2\times2 = 16$

$\pi/3 + \pi/3 + \pi/3 + \pi/3 = 4\pi/3$.

Q9 $z = 3\left(\cos\pi + i\sin\pi\right)$

$\frac{1}{z} = \frac{1}{3}\left(\cos(-\pi) + i\sin(-\pi)\right)$

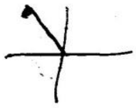
Note.
Tables: $\cos(-\pi) = \cos\pi$ and $\sin(-\pi) = -\sin\pi$.

Correct Ans

or $= \frac{1}{3}\left(\cos\pi - i\sin\pi\right)$

$= \frac{1}{3}(-1 + 0i) = -\frac{1}{3} + 0i$.

Q10 $\quad z = -2 + 2\sqrt{3}\, i$



$$r = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{4 + 12} = 4$$

$$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{-2}\right) = \tan^{-1}(-\sqrt{3}) = -\pi/3$$

$$\Rightarrow \theta = \frac{2\pi}{3}$$

$$\Rightarrow -2 + 2\sqrt{3}i = 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

(a) (i) $z^2 = \left[4\left(\cos\frac{2\pi}{3} + i\sin 2\pi/3\right)\right]^2$

$= 16\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

(ii) $= 16\left(-1/2 + i(-\sqrt{3}/2)\right)$

$= -8 - 8\sqrt{3}\, i$

(b) $z^3 = \left[4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\right]^3$

$4 \times 4 \times 4 = 64$

$\frac{2\pi}{3} + \frac{2\pi}{3} + \frac{2\pi}{3} = \frac{6\pi}{3} = 2\pi$

(i) $= 64\left(\cos 2\pi + i\sin 2\pi\right)$

(ii) $64\left(1 + i(0)\right) = 64 + 0i$

$$\boxed{\underline{N.B} \quad \frac{1}{z} = \bar{z}}$$

$\underline{Q11}$ $z = \cos\theta + i\sin\theta \Rightarrow \bar{z} = \cos\theta - i\sin\theta$.

$$\frac{1}{z} = \cos(-\theta) + i\sin(-\theta)$$

Tables: $\cos(-\theta) = \cos\theta$
$\sin(-\theta) = -\sin\theta$

$$\Rightarrow \frac{1}{z} = \cos\theta - i\sin\theta.$$ QED.

$\underline{Q12}$ $z \cdot \left(\cos\frac{\pi}{4} + i\sin\pi/4\right) = 1$ find z.

$$z = \frac{1}{\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}}$$ $\boxed{\frac{1}{z} = \bar{z}}$

$$z = \cos\frac{\pi}{4} - i\sin\frac{\pi}{4}$$

$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i.$$

$\underline{Q13}$ $z = \cos\theta + i\sin\theta \Rightarrow \frac{1}{z} = \cos\theta - i\sin\theta$

$$z + 1/z = (\cos\theta + i\sin\theta) + (\cos\theta - i\sin\theta)$$
$$= 2\cos\theta + 0i$$
$$= 2\cos\theta$$ Q.E.D.