Ex 4·3

Q1 $x^2+y^2=10 \rightarrow$ centre $(0,0)$ radius $=\sqrt{10}$

$\perp$ dis $3x+y+10=0$ to $(0,0)$

$$= \frac{|3(0)+(0)+10|}{\sqrt{3^2+1^2}} = \frac{10}{\sqrt{10}} = \sqrt{10} = \text{radius}$$

$\Rightarrow$ Is a tangent.

Q2 $(x-3)^2+(y+4)^2=50 \rightarrow$ Centre $(3,-4)$ $r=\sqrt{50}$
$=5\sqrt{2}$

$\perp$ dis $x-y+3=0$ to $(3,-4)$

$$= \frac{|(3)-(-4)+3|}{\sqrt{1^2+1^2}} = \frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2} = \text{radius}$$

$\Rightarrow$ Is a tangent.

Q3 $(x+2)^2+(y-1)^2=16 \rightarrow$ centre $(-2,1)$ $r=4$.

$\perp$ dis $3x-4y-12=0$ to $(-2,1)$

$$= \frac{|3(-2)-4(1)-12|}{\sqrt{3^2+4^2}} = \frac{22}{\sqrt{25}} = \frac{22}{5}$$

$\Rightarrow$ Not a tangent

Q4 To find eqn need centre (given) and radius.
find radius: $\perp$ dis $2x-3y-5=0$ to $(-1,2)$

$$= \frac{|2(-1)-3(2)-5|}{\sqrt{2^2+3^2}} = \frac{13}{\sqrt{13}} = \sqrt{13}$$

$\Rightarrow$ eqn: $(x+1)^2+(y-2)^2=13$

Q5 Eqn ⇒ need centre [Given (2,1)] and radius.

find radius : $\perp$ dis $x-y+5=0$ to $(2,1)$

$$= \frac{|(2)-(1)+5|}{\sqrt{1^2+1^2}} = \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}.$$

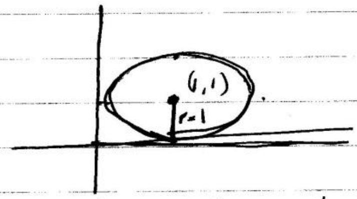
⇒ Eqn: $(x-2)^2 + (y-1)^2 = 18$

Q6 $x^2 + y^2 - 2x - 2y + 1 = 0$

(i) centre $(1,1)$

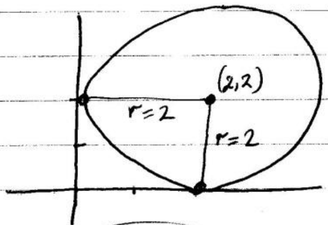
(ii) $\sqrt{1^2+1^2-1} = \sqrt{1} = 1$ = radius

(iii) sketch.



(iv) As radius $=1$ and centre is $(1,1)$, $\perp$ D is to each axis $=1$. hence touches x and y axis.

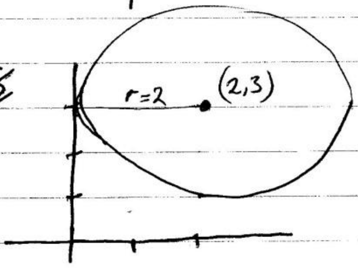
Q7



centre $(2,2)$ radius $=2$.

Eqn: $(x-2)^2 + (y-2)^2 = 4$

Q8



centre $(2,3)$ Touches y axis ⇒ radius $=2$.

Eqn: $(x-2)^2 + (y-3)^2 = 4$

Q9 $x^2 + y^2 - 4x + 6y - 12 = 0$

(i) centre $(2, -3)$ $r = \sqrt{2^2 + (-3)^2 + 12} = \sqrt{25} = 5$

(ii) $3x + 4y - k = 0$ Is tangent $\Rightarrow \perp$ dis $= 5$

$$\frac{|3(2) + 4(-3) - k|}{\sqrt{3^2 + 4^2}} = 5$$

$$\frac{|-6-k|}{\sqrt{25}} = 5$$

$$|-6-k| = 25$$

$-6 - k = -25$
$-k = -19$
$\underline{k = 19}$

or

$-6 - k = 25$
$-k = +31$
$\underline{k = -31}$

OR Sq both sides
$36 + 12k + k^2 = 625$
$k^2 + 12k - 589 = 0$
$(k + 31)(k - 19) = 0$
$k = -31$ or $k = 19$

Q10 (i) Eqn $\perp$ bisector of $(1,2)$ $(0\ -2)$

$m = \frac{-2-2}{0-1} = \frac{-4}{-1} = 4 \Rightarrow \perp m = -\frac{1}{4}$

mid pt $= \left(\frac{1+0}{2}, \frac{2-2}{2}\right) = \left(\frac{1}{2}, 0\right)$

Eqn: $y - 0 = -\frac{1}{4}\left(x - \frac{1}{2}\right)$
$4y = -x + \frac{1}{2}$
$8y = -2x + 1$
$2x + 8y - 1 = 0.$

(ii) $\perp$ Bisector of $(0,-2)$ $(4,-3)$

$m = \frac{-3+2}{4-0} = -\frac{1}{4} \Rightarrow \perp m = 4$

mid pt $= \left(\frac{0+4}{2}, \frac{-2-3}{2}\right) = \left(2, -\frac{5}{2}\right)$

Eqn: $y + \frac{5}{2} = 4(x - 2)$
$y + 5/2 = 4x - 8$
$2y + 5 = 8x - 16$
$8x - 2y - 21 = 0$

(iii) Pt of Intersection: $2x + 8y - 1 = 0$

$$8x - 2y - 21 = 0 \quad (\times 4)$$

$$2x + \cancel{8y} - 1 = 0$$
$$32x - \cancel{8y} - 84 = 0$$
$$34x \quad - 85 = 0$$
$$x = \frac{85}{34} = 5/2.$$

$$8(5/2) - 2y - 21 = 0$$
$$20 - 2y - 21 = 0$$
$$-2y = 1$$
$$y = -1/2$$

Pt of Intersection $(5/2, -1/2)$

(iv) Radius = Dis from $(5/2, -1/2)$ to one of the points.

$(5/2, -1/2)$ to $(0, -2)$

$$= \sqrt{(0 - 5/2)^2 + (-2 + \tfrac{1}{2})^2} = \sqrt{25/4 + 9/4}$$
$$= \sqrt{\frac{34}{4}} = \sqrt{17/2}.$$

(v) Eqn of Circle, Centre $(5/2, -\frac{1}{2})$ $r = \sqrt{17/2}$.

$$(x - 5/2)^2 + (y + \tfrac{1}{2})^2 = 17/2$$

Q11 $x^2+y^2+2gx+2fy+c=0$

$(0,0) \Rightarrow 0^2+0^2+2g(0)+2f(0)+c=0 \Rightarrow \boxed{c=0}$

$(2,0) \Rightarrow (2)^2+(0)^2+2g(2)+2f(0)+0=0$

$4+4g=0$

$4g=-4 \Rightarrow \boxed{g=-1}$

$(3,-1) = (3)^2+(-1)^2+2(-1)(3)+2f(-1)+0=0$

$9+1-6-2f=0$

$-2f=-4$

$\boxed{f=2}$

$\Rightarrow$ Eqn Is: $x^2+y^2+2(-1)x+2(2)y+0=0$

$x^2+y^2-2x+4y=0.$

Q12 $x^2+y^2+2gx+2fy+c=0$

$(0,0) \Rightarrow 0^2+0^2+2g(0)+2f(0)+c=0 \Rightarrow \boxed{c=0}$

$(-2,4) \Rightarrow (-2)^2+(4)^2+2g(-2)+2f(4)+0=0$

$4+16-4g+8f=0$

$\boxed{-4g+8f=-20}$

$(-1,7) \Rightarrow (-1)^2+(7)^2+2g(-1)+2f(7)+0=0$

$1+49-2g+14f=0$

$\boxed{-2g+14f=-50}$

$-4g+8f=-20$

$-2g+14f=-50(x-2)$

$\cancel{-4g}+8f=-20$

$\cancel{4g}+-28f=100$

$-20f=80$

$\boxed{f=-4}$

$-4g+8(-4)=-20$

$-4g=12$

$\boxed{g=-3}$

$\therefore$ Eqn Is:

$x^2+y^2+2(-3)x+2(-4)y+0=0$

$x^2+y^2-6x-8y=0$

Q13 $x^2 + y^2 + 2gx + 2fy + c = 0$

$(3,5) \Rightarrow (3)^2 + (5)^2 + 2g(3) + 2f(5) + c = 0$

$9 + 25 + 6g + 10f + c = 0$

$\boxed{6g + 10f + c = -34}$ A

$(-1,3) \Rightarrow (-1)^2 + (3)^2 + 2g(-1) + 2f(3) + c = 0$.

$1 + 9 - 2g + 6f + c = 0$

$\boxed{-2g + 6f + c = -10}$ B

Centre $(-g, -f)$ is on $x + 2y - 6 = 0$

$\Rightarrow -g + 2(-f) - 6 = 0$

$\boxed{-g - 2f = 6}$ C

A $6g + 10f + \cancel{c} = -34$

$-$B $2g - 6f \cancel{-c} = 10$

$8g + 4f = -24$

C $-g - 2f = 6$ $(\times 2)$

$8g + \cancel{4f} = -24$

$-2g - \cancel{4f} = 12$

$6g = -12$

$\boxed{g = -2}$

$-(-2) - 2f = 6$

$2 - 2f = 6$

$-2f = 4$

$\boxed{f = -2}$

A $6(-2) + 10(-2) + c = -34$

$-12 - 20 + c = -34$

$\boxed{c = -2}$

$\therefore$ Eqn Is

$x^2 + y^2 + 2(-2)x + 2(-2)y - 2 = 0$

$x^2 + y^2 - 4x - 4y - 2 = 0$.

Q14 $x^2 + y^2 + 2gx + 2fy + c = 0$ Centre $(-g, -f)$
lies on x axis $\Rightarrow$ centre is $(-g, 0)$ $\boxed{f = 0}$

$(4,5) \Rightarrow (4)^2 + (5)^2 + 2g(4) + 2(0)(5) + c = 0$

$16 + 25 + 8g + c = 0$

$\boxed{8g + c = -41}$ (A)

$(-2,3) \Rightarrow (-2)^2 + (3)^2 + 2g(-2) + 2(0)(3) + c = 0$

$4 + 9 - 4g + c = 0$

$\boxed{-4g + c = -13}$ (B)

(A) $8g + \cancel{c} = -41$

(B) $\overset{+}{-}4g \, \overset{-}{+}\cancel{c} = \overset{+}{-}13$

$12g = -28$

$g = -28/12$

$\boxed{g = -7/3}$

(B) $-4\left(\frac{-7}{3}\right) + c = -13$

$\frac{28}{3} + c = -13$

$c = -13 - \frac{28}{3}$

$\boxed{c = \frac{-67}{3}}$

$\therefore$ Eqn is: $x^2 + y^2 + 2\left(\frac{-7}{3}\right)x + 2(0)y - \frac{67}{3} = 0$

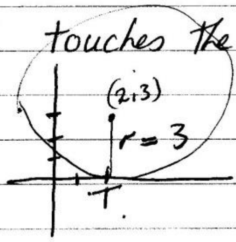
$x^2 + y^2 - \frac{14}{3}x - \frac{67}{3} = 0 \quad (\times 3)$

$3x^2 + 3y^2 - 14x - 67 = 0$.

Q15 $\quad x^2 + y^2 - 4x - 6y + k = 0$

centre $(2, 3)$ $\quad$ radius $= \sqrt{2^2 + 3^2 - k} = \sqrt{13 - k}$

Centre $(2,3)$ and touches the x axis

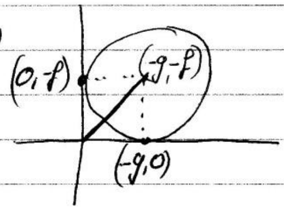


radius $= 3$.

$$\sqrt{13 - k} = 3$$
$$13 - k = 9$$
$$-k = -4$$
$$k = 4$$

Co-ords of $T = (2, 0)$

Q16 (i)



Both g and f are negative
also $g = f$.

(ii) dis $(0,0)$ to $(-g, -f)$ is $3\sqrt{2}$

$$\sqrt{(-g-0)^2 + (-f-0)^2} = 3\sqrt{2}$$
$$\sqrt{g^2 + f^2} = 3\sqrt{2}$$
$$g^2 + f^2 = 18$$

$(0, -f) \Rightarrow 0^2 + f^2 + 0 + 2f(-f) + c = 0$

$$f^2 - 2f^2 + c = 0$$
$$-f^2 + c = 0$$

$(-g, 0) \Rightarrow \quad g^2 + 0 + 2g(-g) + 0 + c = 0$

$g^2 - 2g^2 + c = 0$

$-g^2 + c = 0$

$-f^2 + \cancel{c} = 0$

$+ \; -g^2 - \cancel{+c} = 0$

$-f^2 + g^2 = 0$

$\boxed{\begin{aligned} -f^2 + c &= 0 \\ -(3)^2 + c &= 0 \\ -9 + c &= 0 \\ c &= 9. \end{aligned}}$

$g^2 + \cancel{f^2} = 18$

$g^2 - \cancel{f^2} = 0$

$2g^2 = 18$

$g^2 = 9$

$\boxed{g = 3} \quad \Rightarrow \quad \boxed{f = 3} \quad [\text{as } g = f.]$

$\therefore$ Eqn is: $x^2 + y^2 + 2(3)x + 2(3)y + 9 = 0$

$x^2 + y^2 + 6x + 6y + 9 = 0$

Q17 (i) Geometry: Property of circle
The Perpendicular to a tangent at the point of contact passes through the centre of the circle

(ii) Eqn of $\perp$ to tangent at (4,0)

$3x + 2y - 12 = 0 \qquad m = -3/2 \Rightarrow \perp m = 2/3$

~~Eqn: $y - 0 = -3/2(x-4)$~~

~~$2y = -3x + 12$~~

~~$\boxed{3x + 2y - 12 = 0}$~~

Eqn: $y - 0 = 2/3(x-4)$

$3y = 2x - 8$

$\boxed{2x - 3y - 8 = 0}$

(iii) Eqn of $\perp$ bisector of $A(3,-5)$ $B(4,0)$

$m = \frac{0+5}{4-3} = 5/1 \quad \Rightarrow \perp m = -1/5$

Mid Point : $\left(\frac{3+4}{2}, \frac{-5+0}{2}\right) = \left(7/2, -5/2\right)$

Eqn: $y + 5/2 = -1/5(x - 7/2)$

$5y + 25/2 = -x + 7/2 \quad (\times 2)$

$10y + 25 = -2x + 7$

$2x + 10y + 18 = 0$

$\boxed{x + 5y + 9 = 0}$

(iv) find centre:

$2x - 3y - 8 = 0$

$x + 5y + 9 = 0 \quad (\times -2)$

$\cancel{2x} - 3y - 8 = 0$

$\cancel{-2x} - 10y - 18 = 0$

$-13y - 26 = 0$

$-13y = 26$

$y = -2$

$x + 5(-2) + 9 = 0$

$x - 10 + 9 = 0$

$x = 1$

Centre $(1, -2)$

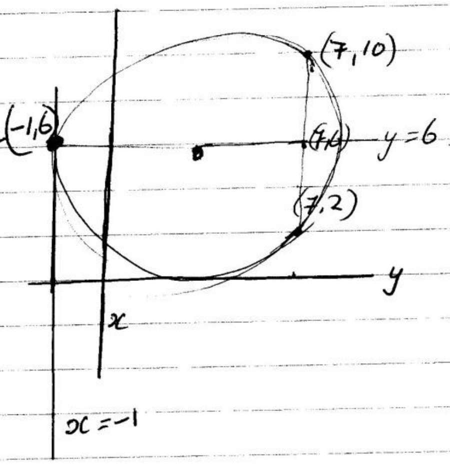
find radius: Distance from $(1,-2)$ to $(4,0)$

$$\sqrt{(4-1)^2+(0+2)^2} = \sqrt{9+4} = \sqrt{13} = \text{radius}$$

(v) Eqn of Circle of centre $(1,-2)$ and radius $= \sqrt{13}$

$$(x-1)^2+(y+2)^2 = 13$$
$$x^2-2x+1+y^2+4y+4-13=0$$
$$x^2+y^2-2x+4y-8=0.$$

Q18



Eqn $\perp$ Bisector of $(7,10)$ and $(7,2)$

$$m = \frac{2-10}{7-7} = \frac{-8}{0} = \text{undf} \quad \perp m = 0$$

$$\text{mid pt} = \left(\frac{7+7}{2}, \frac{10+2}{2}\right) = (7,6)$$

Eqn: $y-6 = 0(x-7)$

$$\boxed{y=6}$$

$$(-1,6)$$

$$\boxed{x=-1}$$

Now have 3 pts on the circle.
$(7,10)$ $(7,2)$ $(-1,6)$

$\perp$ Bisector of $(7,10)$ $(7,2)$ is $y=6$.

$\perp$ Bisector of $(-1,6)$ $(7,2)$

mid pt $\left(\frac{-1+7}{2}, \frac{6+2}{2}\right)$ $(3,4)$

$$m = \frac{2-6}{7+1} = \frac{-4}{8} = \frac{-1}{2} \Rightarrow \perp m = 2$$

Eqn: $y-4 = 2(x-3)$
$$y-4 = 2x-6$$
$$2x-y-2=0.$$

Pt of Intersection is the centre.

$$2x - y - 2 = 0$$
$$y = 6$$

$$2x - 6 - 2 = 0$$
$$2x = 8$$
$$x = 4.$$

Centre $(4, 6)$

radius is dis $(4, 6)$ to $(-1, 6)$ $\Rightarrow r = 5$

eqn is $(x-4)^2 + (y-6)^2 = 25$

$$x^2 - 8x + 16 + y^2 - 12y + 36 = 25$$

$$x^2 + y^2 - 8x - 12y + 27 = 0.$$

Q19 radius $= \sqrt{20}$ contains pt $(-1, 3)$
centre on line $x+y=0$

$$x^2+y^2+2gx+2fy+c=0.$$

$(-1,3) \Rightarrow (-1)^2+(3)^2+2g(-1)+2f(3)+c=0$

$1+9-2g+6f+c=0$

$\boxed{-2g+6f+c=-10}$ (A)

The line $x+y=0$ contains centre $(-g, -f)$

$\Rightarrow -g-f=0$

$\Rightarrow -g=f \Rightarrow \boxed{g=-f}$

$\therefore -2(-f)+6f+c=-10$

$2f+6f+c=-10$

$\boxed{8f+c=-10}$ (B)

radius $= \sqrt{20}$

$\Rightarrow \sqrt{f^2+g^2-c} = \sqrt{20}$ [but $g=-f$]

$\Rightarrow \sqrt{f^2+(-f)^2-c} = \sqrt{20}$

$\sqrt{2f^2-c} = \sqrt{20}$

$\boxed{2f^2-c=20}$ (C)

(B) $8f+c=-10$

(C) $2f^2-c=20$

$2f^2+8f=10$

$f^2+4f-5=0$

$(f+5)(f-1)=0$

$f=-5$ or $f=1$

At $f = -5$ $\Rightarrow$ $g = 5$

(A) $-2(5) + 6(-5) + c = -10$

$-10 - 30 + c = -10$

$c = 30$

$x^2 + y^2 + 2(5)x + 2(-5)y + 30 = 0$

$x^2 + y^2 + 10x - 10y + 30 = 0$ Eqn 1

At $f = 1$ $\Rightarrow$ $g = -1$

(A) $-2(-1) + 6(1) + c = -10$

$2 + 6 + c = -10$

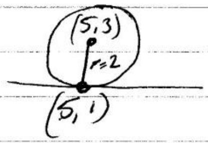
$c = -18$

$x^2 + y^2 + 2(-1)x + 2(1)y - 18 = 0$

$x^2 + y^2 - 2x + 2y - 18 = 0$ Eqn 2

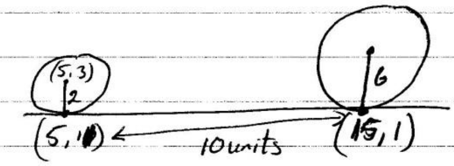
Q20 $(x-5)^2+(y-3)^2=4$ centre $(5,3)$ radius $=2$.

(i)



$\Rightarrow$ ground is line $y=1$

(ii) rear wheel radius $=6$.



$\Rightarrow$ centre $=(15,7)$

eqn: Centre $(15,7)$ $r=6$

$$(x-15)^2+(y-7)^2=36$$

(iii) 2 units to left $\Rightarrow$ new centre $(13,1)$

radius $=6$

New Eqn: $(x-13)^2+(y-7)^2=36$