Ex 4.5

$$S_n = \frac{a(1-r^n)}{1-r}$$

Q1 find S_{10} of $2+6+18+54 \ldots$

$a = 2 \quad r = 3$

$$S_{10} = \frac{2(1-3^{10})}{1-3} = \frac{-\cancel{2}(1-3^{10})}{-\cancel{2}} = -1+3^{10}$$

$$= 59{,}048.$$

Q2 find n° of terms. & hence S_n.

$1024 + 512 + 256 + \cdots 32$

$a = 1024 \qquad r = \frac{1}{2}$

$T_n = ar^{n-1}$

$32 = 1024\left(\frac{1}{2}\right)^{n-1}$

$\frac{32}{1024} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{32} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{2^5} = \frac{1}{2^{n-1}}$

$2^{n-1} = 2^5$

$n-1 = 5$

$n = 6.$

$\frac{1}{32} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{32} = \frac{1}{2^{n-1}}$

$32 = 2^{n-1}$

$\log 32 = n-1 \log 2$

$\frac{\log 32}{\log 2} = n-1$

$5 = n-1$

$6 = n$

$$\Rightarrow S_6 = \frac{1024\left(1-\frac{1}{2}^6\right)}{1-\frac{1}{2}} = 2048\left[1-\left(\frac{1}{2}\right)^6\right] = 2016.$$

Q3 S_8 of $1+2+4+8+\cdots$

$a = 1 \quad r = 2.$

$$S_8 = \frac{1(1-2^8)}{1-2} = -1(1-2^8) = 2^8-1 = 255$$

Q4 S_{10} of $32+16+8+\cdots$

$a = 32 \quad r = \tfrac{1}{2}.$

$$S_{10} = \frac{32(1-\frac{1}{2}^{10})}{1-\frac{1}{2}} = 64(1-\tfrac{1}{2}^{10}) = 63.9375$$

Q5 S_6 of $4-12+36-108+\cdots$

$a = 4 \quad r = \frac{-12}{4} = -3.$

$$S_6 = \frac{4(1-(-3)^6)}{1-(-3)} = (1-(-3)^6) = -728$$

Q6 No of Terms and hence S_n.

$729 - 243 + 81 - \cdots - \tfrac{1}{3}.$

$a = 729 \quad r = \frac{-243}{729} = -\tfrac{1}{3}$

$T_n = ar^{n-1}$

$-\frac{1}{3} = 729\left(-\frac{1}{3}\right)^{n-1}$

$\frac{-1}{2187} = \left(-\frac{1}{3}\right)^{n-1}$

$-\frac{1}{3^7} = \frac{-1}{3^{n-1}}$

$7 = n-1$

$8 = n$

$$S_8 = \frac{729(1-(-\frac{1}{3})^8)}{1-(-\frac{1}{3})} = 546.75(1-(-\tfrac{1}{3})^8)$$

$= 5\cancel{4}6.67.$

OR

$\frac{-1}{2187} = \frac{-1}{3^{n-1}}$

$\log 2187 = n-1 \log 3$

$\frac{\log 2187}{\log 3} = n-1$

$7 = n-1$

$8 = n.$

Q7 first 3 terms of $\sum_{r=1}^{6} 4^r$ hence S_n.

$$T_1 = 4^1 = 4$$
$$T_2 = 4^2 = 16 \qquad \Rightarrow a = 4 \quad r = 4.$$
$$T_3 = 4^3 = 64$$

$n=6$ $$S_6 = \frac{4(1-4^6)}{1-4} = -\frac{4}{3}(1-4^6) = 5460$$

Q8 $$\sum_{r=1}^{8} 2\times 3^r = 2\times 3^1 + 2\times 3^2 + 2\times 3^3 + \cdots$$
$$= 6 + 18 + 54 + \cdots$$

$$\Rightarrow a = 6 \quad r = 3.$$

$n = 8$ $$S_8 = \frac{6(1-3^8)}{1-3} = -3(1-3^8) = 19680.$$

Q9 $$\sum_{r=1}^{10} 6\times\left(\tfrac{1}{2}\right)^r = 6\times\left(\tfrac{1}{2}\right)^1 + 6\times\left(\tfrac{1}{2}\right)^2 + 6\times\left(\tfrac{1}{2}\right)^3 + \cdots$$
$$3 + 1.5 + 0.75 + \cdots$$

$$a = 3 \quad r = \tfrac{1}{2}.$$

$n = 10$ $$S_n = \frac{3(1-\frac{1}{2}^{10})}{1-\frac{1}{2}} = 6\left(1-\left(\tfrac{1}{2}\right)^{10}\right) = 5.994.$$

Q10

(i) $0.\dot{7} = 0.777\cdots = \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \cdots$

$a = 7/10 \quad r = \frac{1}{10}.$

$\lim_{n\to\infty} S_n = \frac{a}{1-r} = \frac{7/10}{1-\frac{1}{10}} = \frac{7/10}{\frac{9}{10}} = 7/9$

(ii) $0.\dot{3}\dot{5} = 0.353535\cdots$

$= \frac{35}{100} + \frac{35}{10000} + \frac{35}{1000000} \cdots .$

$\Rightarrow a = \frac{35}{100} \quad r = \frac{1}{100}$

$\lim_{n\to\infty} S_n = \frac{a}{1-r} = \frac{35/100}{1-1/100} = \frac{35}{100} \times \frac{100}{99} = \frac{35}{99}$

(iii) $0.2\dot{3} = 0.23333\cdots$

$= 0.2 + \left[\frac{3}{100} + \frac{3}{1000} + \frac{3}{10000} + \cdots\right.$

$= 0.2 + \left[a = 3/100 \quad r = \frac{3}{1000} \times \frac{100}{3} = \frac{1}{10} \right]$

$\left[\lim_{n\to\infty} S_n = \frac{3/100}{1-\frac{1}{10}} = \frac{3}{100} \times \frac{10}{9} = \frac{3}{90} = \frac{1}{30}\right]$

$\Rightarrow 0.2 + 1/30 = 7/30.$

(iv) $0.\dot{3}7\dot{0} = 0.370\,370\ldots$

$= \frac{370}{1000} + \frac{370}{1000000} + \ldots$

$a = \frac{370}{1000} \qquad r = \frac{\cancel{370}}{1000000} \times \frac{1000}{\cancel{370}} = \frac{1}{1000}.$

$\lim_{n \to \infty} S_n = \frac{\frac{370}{1000}}{1 - \frac{1}{1000}} = \frac{370}{1000} \times \frac{1000}{999} = \frac{370}{999} = \frac{10}{27}$

(v) $0.1\dot{6}\dot{2} = 0.1626262$

$= 0.1 + \left[\frac{62}{1000} + \frac{62}{100000} + \ldots\right]$

$= 0.1 + \left[a = \frac{62}{1000} \quad r = \frac{62}{100000} \times \frac{1000}{62} = \frac{1}{100}\right]$

$\lim_{n \to \infty} S_n = \frac{62/1000}{1 - 1/100} = \frac{62}{1000} \times \frac{100}{99} = \frac{62}{990}$

$= 0.1 + \left[\frac{62}{990}\right] = \frac{161}{990}$

(vi) $0.3\dot{2}\dot{1} = 0.3212121\ldots$

$= 0.3 + \left[\frac{21}{1000} + \frac{21}{100000} + \ldots\right]$

$\left[a = \frac{21}{1000} \quad r = \frac{\cancel{21}}{100000} \times \frac{1000}{\cancel{21}} = \frac{1}{100}\right]$

$\lim_{n \to \infty} S_n = \frac{\frac{21}{1000}}{1 - \frac{1}{100}} = \frac{21}{1000} \times \frac{100}{99} = \frac{21}{990}$

$= \frac{7}{330}.$

$\Rightarrow 0.3 + \left[\frac{7}{330}\right] = \frac{53}{165}.$

Q11

S_n of $1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots \left(\frac{1}{2}\right)^{n-1}$.

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{1\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}} = 2\left(1-\left(\frac{1}{2}\right)^n\right)$$

$$= 2 - \frac{2^1}{2^n}$$

$$= 2 - \frac{1}{2^{n-1}} \text{ or } 2 - 2^{1-n}.$$

$$S_\infty = \lim_{n\to\infty} S_n = \frac{1}{1-\frac{1}{2}} = 2.$$

Least Value of n such That $S_\infty - S_n < 0.001$

$$S_\infty - S_n < 0.001$$

$$2 - \left[2 - \frac{1}{2^{n-1}}\right] < 0.001$$

$$\cancel{2} - \cancel{2} + \frac{1}{2^{n-1}} < 0.001$$

$$2^{n-1} > \frac{1}{0.001}$$

$$2^{n-1} > 1000$$

$$n-1 \log 2 > \log 1000$$

$$n-1 > \frac{\log 1000}{\log 2}$$

$$n-1 > 9.96578$$

$$n > 10.96578$$

$$n = 11$$