

Exercise 5.5

Q1

$$(i) \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

$$(ii) \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

$$(iii) \tan^{-1}(1) = 45^\circ$$

$$(iv) \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$$

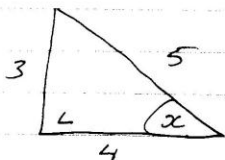
$$(v) \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -60^\circ$$

$$(vi) \tan^{-1}(-1) = -45^\circ$$

$$(vii) \cos^{-1}\left(-\frac{1}{2}\right) = -60^\circ$$

$$(viii) \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -30^\circ$$

Q2
 (i) $\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

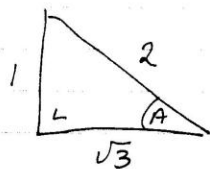


$$\sin^{-1}\left(\frac{3}{5}\right) = x$$

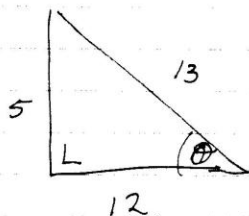
$$\Rightarrow \sin x = \frac{3}{5} \quad \frac{O}{H}$$

$$\Rightarrow \tan x = \frac{3}{4} \quad \frac{O}{A}$$

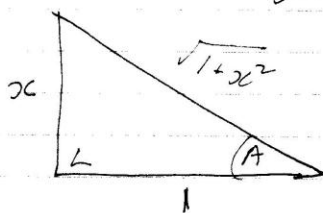
(ii) $\sin^{-1}\left(\frac{1}{2}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$



(iii) $\sin^{-1}\left(\frac{5}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)$



(iv) $\tan^{-1}(x) = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$

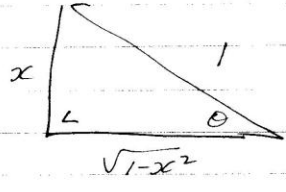


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Q3

(i) $\sin(\sin^{-1} x)$

↓
Draw This Angle



$$\sin(\text{Angle}) = \frac{O}{H}$$

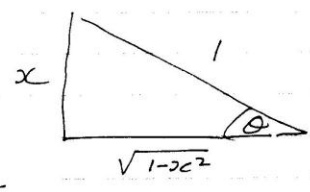
$$\sin(\sin^{-1} x) = \frac{x}{1} = x$$

$$1^2 = x^2 + y^2$$
$$\sqrt{1-x^2} = y$$

(ii)

$$\cos(\sin^{-1} x)$$

↓
Draw This Angle



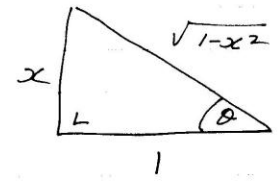
$$\cos(\sin^{-1} x) = \frac{A}{H} = \frac{\sqrt{1-x^2}}{1}$$

$$= \sqrt{1-x^2}$$

(iii)

$$\sin(\tan^{-1} x)$$

↓
Draw This Angle



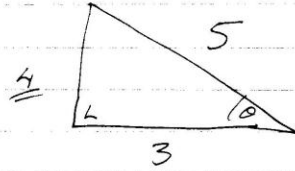
$$\sin(\tan^{-1} x) = \frac{O}{H}$$

$$= \frac{x}{\sqrt{1+x^2}}$$

Q4
 (i) $\sin(\cos^{-1} \frac{3}{5})$
 ↓ Draw

$$\sin(\cos^{-1} \frac{3}{5}) = \frac{O}{H}$$

$$= \frac{4}{5}$$



(ii) $\cos(\tan^{-1} 1)$

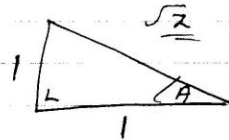
$$\cos(45) = \frac{1}{\sqrt{2}}$$

OR

$\cos(\tan^{-1} 1)$
 Draw

$$\cos(\tan^{-1} 1) = \frac{A}{H}$$

$$= \frac{1}{\sqrt{2}}$$

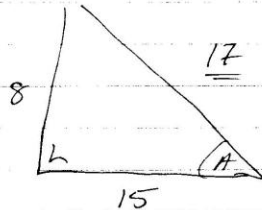


$$x^2 = 1^2 + 1^2$$

$$x = \sqrt{2}$$

(iii) $\sin(\tan^{-1} \frac{8}{15})$

$$\sin A = \frac{8}{17}$$



$$8^2 + 15^2 = x^2$$

$$289 = x^2$$

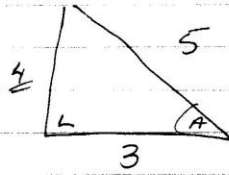
$$17 = x$$

Q5 Evaluate

(i) $\sin\left(2 \cos^{-1} \frac{3}{5}\right)$

Draw

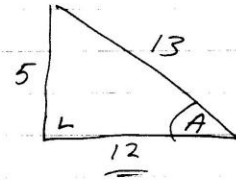
let $\cos^{-1} \frac{3}{5} = A$



$$\begin{aligned}\sin 2A &= 2 \sin A \cos A \\ &= 2 \left(\frac{4}{5}\right) \left(\frac{3}{5}\right) = \frac{24}{25}\end{aligned}$$

(ii) $\cos\left(2 \sin^{-1} \frac{5}{13}\right)$

let $A = \sin^{-1} \frac{5}{13}$ (Draw)



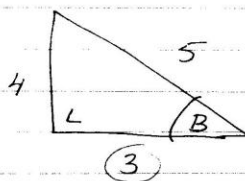
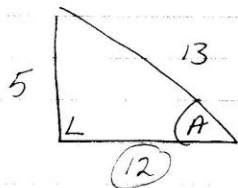
$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 \\ &= \frac{144}{169} - \frac{25}{169} \\ &= \frac{119}{169}\end{aligned}$$

$$\begin{aligned}13^2 &= 5^2 + x^2 \\ x &= 12\end{aligned}$$

Q6 $\sin(A+B) = \sin A \cos B + \cos A \sin B$

(i) $\sin \left[\sin^{-1} \left(\frac{5}{13} \right) + \sin^{-1} \left(\frac{4}{5} \right) \right] = \frac{63}{65}$

Let $\sin^{-1} \frac{5}{13} = A$ and $\sin^{-1} \frac{4}{5} = B$



$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\left(\frac{5}{13} \right) \left(\frac{3}{5} \right) + \left(\frac{12}{13} \right) \left(\frac{4}{5} \right)$

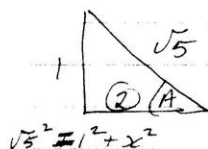
$\frac{3}{13} \frac{15}{65} + \frac{48}{65}$

$= \frac{63}{65}$ RHS

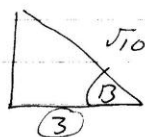
(ii) $\sin \left[\sin^{-1} \frac{1}{\sqrt{5}} + \sin^{-1} \frac{1}{\sqrt{10}} \right] = \frac{1}{\sqrt{2}}$

$A = \sin^{-1} \frac{1}{\sqrt{5}}$

$B = \sin^{-1} \frac{1}{\sqrt{10}}$



$\sqrt{5}^2 = 1^2 + 2^2$



$\sqrt{10}^2 = 1^2 + 3^2$
 $3 = x$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

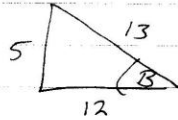
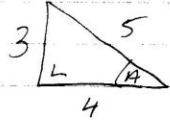
$\left(\frac{1}{\sqrt{5}} \right) \left(\frac{3}{\sqrt{10}} \right) + \left(\frac{2}{\sqrt{5}} \right) \left(\frac{1}{\sqrt{10}} \right)$

$\frac{3}{5\sqrt{2}} + \frac{2}{5\sqrt{2}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$

RHS

Q7 $\tan \left[\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{5}{13} \right]$

Let $A = \sin^{-1} \frac{3}{5}$ and $B = \sin^{-1} \frac{5}{13}$



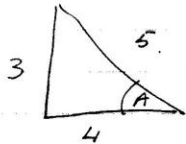
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\left(\frac{3}{4}\right) + \left(\frac{5}{12}\right)}{1 - \left(\frac{3}{4}\right)\left(\frac{5}{12}\right)} = \frac{\frac{9+5}{12}}{1 - \frac{15}{48}} = \frac{\frac{14}{12}}{\frac{33}{48}}$$

$$= \frac{14}{12} \times \frac{48}{33} = \frac{56}{33}$$

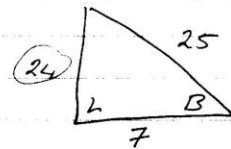
Q8 Prove $\sin \left(2 \tan^{-1} \frac{3}{4} \right) = \sin \left(\cos^{-1} \frac{7}{25} \right)$

Let $A = \tan^{-1} \frac{3}{4}$



$$\begin{aligned} \sin 2A &= 2 \sin A \cos A \\ &= 2 \left(\frac{3}{5}\right) \left(\frac{4}{5}\right) \\ &= \frac{24}{25} \end{aligned}$$

Let $B = \cos^{-1} \frac{7}{25}$



$$\sin B = \frac{24}{25}$$

Both are $\frac{24}{25}$.