Ex 7.9
Q1 (i) $\begin{aligned} & \log _{2} 4=x \\ & 2^{x}=4\end{aligned}$
(ii)

$$
2^{x}=2^{2}
$$

$$
x=2
$$

$$
\begin{gathered}
\log _{3} 81=x \\
3^{x}=81 \\
3^{x}=3^{4} \\
x=4
\end{gathered}
$$

(iii)

$$
\begin{gathered}
\log _{10} 1000=x \\
10^{x}=1000 \\
10^{x}=10^{3} \\
x=3
\end{gathered}
$$

(iv)

$$
\begin{gathered}
\log _{2} 64=x \\
2^{x}=64 \\
2^{x}=2^{6} \\
x=6
\end{gathered}
$$

Q2
(i) $\begin{aligned} \log _{8} 16 & =x \\ 8^{x} & =16\end{aligned}$

$$
\begin{aligned}
& 8^{x}=16 \\
& 2^{3 x}=2^{4}
\end{aligned}
$$

(ii)

$$
\begin{aligned}
\log _{9} 27 & =x \\
9^{x} & =27 \\
3^{2 x} & =3^{3} \\
2 x & =3 \\
x & =3 / 2
\end{aligned}
$$

(iii) $\log _{16} 32=x$

$$
\begin{aligned}
& 16^{x}=32 \\
& 2^{4 x}=2^{5} \\
& 4 x=5 \\
& x=5 / 4
\end{aligned}
$$

(iv) $\log _{\frac{1}{2}} 8=x$

$$
\begin{gathered}
\left(\frac{1}{2}\right)^{x}=8 \\
2^{-x}=2^{3} \\
x=-3
\end{gathered}
$$

(v) $\log _{\frac{1}{3}} 81=x$

$$
\begin{aligned}
\left(\frac{1}{3}\right)^{x} & =81 \\
3^{-x} & =3^{4} \\
x & =-4
\end{aligned}
$$

Qu
(i) $\log _{\frac{1}{3}} 27=x$
(ii) $\log _{\sqrt{2}} 4=x$

$$
\begin{gathered}
\left(\frac{1}{3}\right)^{x}=27 \\
3^{-x}=3^{3} \\
x=-3
\end{gathered}
$$

$$
\begin{aligned}
& \sqrt{2}=4 \\
& 2^{k^{x} x}=2^{2} \\
& x=4 .
\end{aligned}
$$

(iii)

$$
\begin{aligned}
& \log _{8} x=2 \\
& 8^{2}=x \\
& 64-6
\end{aligned}
$$

(iv)

$$
\begin{gathered}
\log _{64} x=\frac{1}{2} \\
64^{2}=x \\
\sqrt{64}=x \\
8=x
\end{gathered}
$$

Q4 (i) $\log _{2} x=-1$

$$
\begin{aligned}
& 2^{-1}=x \\
& \frac{1}{2}=x
\end{aligned}
$$

(iii)

$$
\begin{aligned}
\log _{x} 2 & =2 \\
x^{2} & =2 \\
x & =\sqrt{2}
\end{aligned}
$$

05

$$
\text { (i) } \begin{gathered}
\log _{4} 2+\log _{4} 32 \\
\Rightarrow \log _{4}(2 \times 32)=x \\
\Rightarrow \log _{41} 64=x \\
4^{x}=64 \\
4^{x}=4^{3} \\
x=3
\end{gathered}
$$

(ii)

$$
\begin{gathered}
\log _{3} \sqrt{27}=x \\
3^{x}=\sqrt{27} \\
3^{x}=3^{3 / 2} \\
x=3 / 2
\end{gathered}
$$

(iv)

$$
\begin{gathered}
\log _{2}(0.5)=x \\
2^{x}=0.5 \\
2^{x}=1 / 2 \\
2^{x}=2^{-1} \\
x=-1
\end{gathered}
$$

(ii) $\log _{6} 9+\log _{6} 8-\log _{6} 2$

$$
\begin{array}{r}
\Rightarrow \log _{6} \frac{(9 \times 8)}{2} \\
\Rightarrow \log _{6} 36=x \\
6^{x}=36 \\
x=2
\end{array}
$$

(95 (iii) $\quad \log _{6} 4+2 \log _{6} 3$

$$
\begin{gathered}
\Rightarrow \log _{6} 4+\log _{6} 9 \\
\Rightarrow \log _{6}(4 \times 9) \\
\Rightarrow \log _{6} 36=x \\
6^{x}=36 \\
x=2
\end{gathered}
$$

Q26 (i)

$$
\text { i) } \begin{aligned}
& \log _{3} 2+2 \log _{3} 3-\log _{3} 18 \\
\Rightarrow & \log _{3} 2+\log _{3} 9-\log _{3} 18 \\
\Rightarrow & \log _{3} \frac{(2 \times 9)}{18} . \\
\Rightarrow & \log _{3} 1=0
\end{aligned}
$$

(ii) $\log _{8} 72-\log _{2} \frac{4}{8}$

$$
\begin{gathered}
\Rightarrow \log _{8} \frac{72}{9 / 8)} \Rightarrow \log _{8}\left(\frac{72 \times 8}{9}\right) \\
\Rightarrow \log _{8} 64=x \\
8^{x}=64 \\
x=2 .
\end{gathered}
$$

Q1 $\log _{3} 5=9$.
(i) $\begin{aligned} \log _{3} 15 \Rightarrow \log _{3}(3 \times 5) & \Rightarrow \log _{3} 3+\log _{3} 5 \\ & =1+a .\end{aligned}$
(ii)

$$
\begin{aligned}
\log _{3}\left(\frac{5}{3}\right) & \Rightarrow \log _{3} 5-\log _{3} 3 \\
& =a-1
\end{aligned}
$$

$$
\log _{3} 5=a
$$

Q7 (iii)

$$
\text { ii) } \begin{aligned}
& \log _{3}\left(8 \frac{1}{3}\right) \Rightarrow \log _{3}\left(\frac{25}{3}\right) \Rightarrow \log _{3} 25-5^{(5+5)}-\log _{3} 3 \\
\Rightarrow & \log _{3} 5+\log _{3} 5-\log _{3} 3 \\
\Rightarrow & 2 a-1
\end{aligned}
$$

(iv) $\left.\log _{3}\left(\frac{255}{27}\right)_{3 \times 3 / 3}^{25}\right) \Rightarrow \log _{3} 5+\log _{3} 5-\left[\log _{3} 3+\log _{3} 3+\log _{3} 3\right]$

$$
\Rightarrow 2 a-3
$$

(or

$$
\begin{gathered}
\log _{3} \frac{25}{27} \Rightarrow \log _{3} 25-\log _{3} 27 \\
\log _{3} 5^{2}=\log _{3} 3^{3} \\
2 \log _{3} 5=3 \log _{3} 3 \\
2 a-3(1) \\
2 a-3
\end{gathered}
$$

(v) $\log _{3} 75 \Rightarrow \log _{3}(25 \times 3) \Rightarrow 2 \log _{3} 5+\log _{3} 3$
F) Q8 (i) $200=2^{\circ}$

$$
\begin{aligned}
& \text { U8(1) } 200=2 \\
& \log 200=\log 2^{x} \\
& \log 200=x \log 2 \\
& \frac{\log 200}{}=x \\
&\left(\text { calculator) } \log _{7 \cdot 64} 2\right.=x
\end{aligned}
$$

$$
\text { (ii) } 5^{x}=500
$$

$$
\begin{aligned}
& \log 5^{20}=\log 500 \\
& x \log 5=\log 500
\end{aligned}
$$

$$
x \log 5=\log 500
$$

$$
x=\frac{\log 500}{\log 5}
$$

$$
x=3.86
$$

(88 (iii) $\quad 3^{x+1}=25$

$$
\begin{gathered}
\log 3^{x+1}=\log 25 \\
(x+1) \log 3=\log 25 \\
x+1=\frac{\log 25}{\log 3} \\
x+1=2.9299 \\
x=1.93
\end{gathered}
$$

(iv)

$$
\begin{gathered}
(2 x+3) \log 5=\log 51 \\
2 x+3=\frac{\log 51}{\log 5} \\
2 x+3=2.443 \\
2 x=-0.557 \\
x=-0.279
\end{gathered}
$$

Qq $\quad y=2^{x-1}+3$
(i)

$$
\begin{aligned}
& 2^{x-1}=y-3 \\
& \log 2^{x-1}=\log (y-3) \\
& x-1 \log 2=\log (y-3) \\
& x-1=\frac{\log (y-3)}{\log 2} \\
& x=\frac{\log (y-3)}{\log 2}+1
\end{aligned}
$$

(ii) $\quad y=8$

$$
\begin{aligned}
x & =\frac{\log (8-3)}{\operatorname{tog} 2}+1 \\
x & =\frac{\log 5}{\log 2}+1 \\
x & =2 \cdot 3279+1 \\
x & =3 \cdot 3219
\end{aligned}
$$

Q10 $\log _{10} x=1+a$
Show $x y=100$
$\log _{10} y=1-a$.

$$
\begin{aligned}
& 10^{1+a b}=x \quad 10^{1-a}=y \\
\Rightarrow x y & =\left(10^{1+a}\right)\left(10^{1-a}\right) \quad \text { mut } \Rightarrow \text { add powers } \\
x y & =10^{1+a+1-a} \\
x y & =10^{2} \\
x y & =100
\end{aligned}
$$

(11) $p=\log _{a}\left(\frac{21}{4}\right) \quad 2 q=\log _{4}\left(\frac{7}{3}\right) \quad r=\log _{4}\left(\frac{1}{2}\right)$

Show $p+q=2 r$

$$
\begin{aligned}
\Rightarrow & \log _{a}\left(\frac{21}{4}\right)+\log _{a}\left(\frac{7}{3}\right)=2 \log _{a}\left(\frac{7}{2}\right) \\
\Rightarrow & \log _{a} 21-\log _{a} 4+\log _{a} 7-\log _{a} 3=2 \log _{a}\left(\frac{7}{2}\right) \\
& \log _{a}(3 \times 7)-\log _{a}(2 \times 2)+\log _{a} 7-\log _{a} 3 \\
& \log _{a} 3+\log _{a} 7-\left(\log _{a} 2+\log _{a} 2\right)+\log _{a} 7-\log _{a} 3 \\
& 2 \log _{a} 7-2\left(\log _{a} 2\right) \\
& 2\left(\log _{a} 7-\log _{a} 2\right) \\
& 2 \log _{a}\left(\frac{7}{2}\right)
\end{aligned}
$$

Hence $p+q=2 r$

Q12 $\log _{a} x=4$
(i) $\log _{a} x^{2} y$

$$
\begin{aligned}
& \Rightarrow \log _{a} x^{2}+\log _{a} y \\
& =2 \log _{a} x+\log _{a} y \\
& =2(4)+5 \\
& =8+5 \\
& =13
\end{aligned}
$$

(iii)

$$
\begin{aligned}
& \log _{a} \frac{\sqrt{x}}{y} \\
& \log _{a} x^{\frac{1}{2}}-\log _{a} y \\
& \frac{1}{2} \log _{a} x-5 \log _{a} y \\
& \frac{1}{2}(4)-5=-3 \\
& 2-5=-3
\end{aligned}
$$

$$
\text { Q13 } \begin{aligned}
\log _{25} x & =\frac{1}{2} \log _{5} x \\
\log _{25} x & =\frac{\log _{5} x}{\log _{5}-25} \\
& =\frac{\log _{5} x}{\log _{5} 5^{2}} \\
& =\frac{\log _{5} x}{2\left(\log _{5} 5\right)}=\frac{\log _{5} x}{2}=\frac{1}{2} \log _{5} x
\end{aligned}
$$

Q14 (i) $\log _{10} 4=0.60^{2}$
(ii) $\log _{10} 27=1.43$
(iii) $\log _{10} 356=2.55$
(iv) $\log _{10} 5600=3.75$
(v) $\log _{12} 29000=4 \cdot 46$
(vi) $\log _{10} 350,000 \div 5.54$
(vii) $\log _{10} 3,870,000=6.59$

Q15 $\quad \log _{10} x=3.123$

$$
\begin{array}{rlr}
10^{3 \cdot 123}=x & 10^{3}=1000 & \text { (hin) } \\
& 10^{4}=10,000 & \text { (naxe) }
\end{array}
$$

(Q16 $\log _{3} 15-\log _{2} 5$

$$
\begin{aligned}
& \frac{\log _{10} 15}{\operatorname{tg}_{0} 3}-\frac{\log _{1} 5}{\operatorname{tg} 2} \\
& \begin{aligned}
2.46497-2.3219 & =0.14307 \\
& =0.143
\end{aligned}
\end{aligned}
$$

Q17 (1) $\log _{27} 81=\frac{\log _{3} 81}{\log _{3} 27}=\frac{4}{3}$
(ii) $\log _{32} 8=\frac{\log _{2} 8}{\frac{\log _{2}}{} 32}=\frac{3}{5}$

Q $18 \log _{6} a=\frac{1}{\log _{a} b}$ N/3. lito
change $\log _{b} a$ to base $a$

$$
\begin{equation*}
\frac{\log _{a} a}{\log _{a} b}=\frac{1}{\log _{a} b} \tag{CED.}
\end{equation*}
$$

Q19 If $x>0$ show $\frac{1}{\log _{2} x}+\frac{1}{\log _{3} x}+\frac{1}{\log _{3} x}=\frac{1}{\log _{30} x}$
Using fact established in Q18.

$$
\begin{align*}
& \log _{x} 2+\log _{2} 3+\log _{2} 5 \\
& =\log _{x}(2 \times 3 \times 5) \\
& =\log _{x} 30=\frac{1}{\log _{3} x} \tag{COED}
\end{align*}
$$

Q20 $\log _{,} P=\log _{2} 2+3 \log _{-} q$ express Pinterms

$$
\begin{aligned}
\log _{r} P & =\log _{\Gamma} 2+\log _{\Gamma} q^{3} \\
\log _{r} P & =\log _{r} 2 q^{3} \\
P & =2 q^{3}
\end{aligned}
$$

Q21 $\log _{3} a+\log _{9} a=\frac{3}{4} \quad$ find $a$

$$
\begin{aligned}
& \log _{3} a+\frac{\log _{3} a}{\log _{3} a}=\frac{3}{4} \\
& \log _{3} a+\frac{\log _{3} a}{\log _{3} 3^{2}}=\frac{3}{4} \\
& \log _{3} a+\frac{\log _{3} a}{2 \log _{3} 3}=3 / 4 \\
& \log _{3} a+\frac{\log _{3} a}{2}=\frac{3}{4} \quad \text { (mult by } 2 \text { ) } \\
& 2 \log _{3} a+\log _{3} a=3 / 2 \\
& 3 \log _{3} a=3 / 2 \\
& \log _{3} a=\frac{1}{2} \\
& 3^{\frac{1}{2}}=a \\
& \sqrt{3}=a .
\end{aligned} \quad \text { (Dinde by } 3 \text { ) } \quad . \quad \text {. }
$$

Q 22

$$
\begin{gathered}
3 \ln 41.5-\ln 250 \\
3(3.7257)-5.5215 \\
11.177-5.5215 \\
=5.6555 \\
=5.66
\end{gathered}
$$

$$
[f 5+\text { eg 6.] }
$$

## Solving Logarithmic Equations

- When solving log equations, ensure that each term has the same base, if not the change of base rule must be used first.
- If no base given, the equation is true for all bases.
- If $\log _{a} b=\log _{a} c$, then $b=c$
- If $\log _{a} b=k$, then $b=a^{k}$
- Check all answers to ensure they do not produce logs of negative numbers as these are not defined.


## Example 5

Solve the equation $2 \log _{3} x-\log _{3}(18-x)=1$.

## Example 6

Solve the equation $\log _{3} x+3 \log _{x} 3=4$.

Q23

$$
\begin{aligned}
& \log _{2}(x-2)+\log _{2} x=3 \\
& \log _{2}(x-2)(x)=3 \\
& \log _{2}\left(x^{2}-2 x\right)=3 \\
& 2^{3}=x^{2}-2 x \\
& 8=x^{2}-2 x \\
& 0=x^{2}-2 x-8 \\
& 0=(x-4)(x+2) \\
& x=4 \quad x-2 \text { not Falud. }
\end{aligned}
$$

Ars: $x=4$

Q 24

$$
\begin{aligned}
& \log _{10}\left(x^{2}+6\right)-\log _{10}\left(x^{2}-1\right)=1 \\
& \log _{10}\left(\frac{x^{2}+6}{x^{2}-1}\right)=1 \\
& 10=\frac{x^{2}+6}{x^{2}-1} \\
& 10\left(x^{2}-1\right)=x^{2}+6 \\
& 10 x^{2}-10=x^{2}+6 \\
& 9 x^{2}-16=0 \\
& (3 x+4)(3 x-4)=0 \\
& 3 x=-4
\end{aligned}
$$

$x$ is always squaved
$\therefore$ Ans: $x= \pm 4 / 3$

Q25 $\log 2 x-\log (x-7)=\log 3$

$$
\begin{gathered}
\log \left(\frac{2 x}{x-7}\right)=\log 3 \\
\frac{2 x}{x-7}=3 \\
2 x=3 x-21 \\
21=x
\end{gathered}
$$

Q26 $\log (2 x+3)+\log (x-2)=2 \log x$

$$
\begin{aligned}
& \log (2 x+3)(x-2)=\log x^{2} \\
& (2 x+3)(x-2)=x^{2} \\
& 2 x^{2}-4 x+3 x-6=x^{2} \\
& x^{2}-x-6=0 \\
& (x-3)(x+2)=0 \\
& x=3 \text { ox=-2 } \rightarrow \text { not Talid. } \\
& \Rightarrow x=3 \text { Ans }
\end{aligned}
$$

(Q27) $\log _{10}(17-3 x)+\log _{10} x=1$

$$
\begin{gathered}
\log _{10}(17-3 x)(7 x)=1 \\
11^{\prime}=(17-3 x)(x) \\
10=17 x-3 x^{2} \\
3 x^{2}-17 x+10=0 \\
(3 x-2)(x-5)=0 \\
x=2 / 3 \quad x=5
\end{gathered}
$$

Q28

$$
\begin{aligned}
& \log _{10}\left(x^{2}-4 x-11\right)=0 \\
& 10^{\circ}=x^{2}-4 x-11 \\
& 1=x^{2}-4 x-11 \\
& 0=x^{2}-4 x-12 \\
& 0=(x-6)(x+2) \\
& x=6 \quad x=-2 \quad(-2)^{2}-4(-2)-11=1 \text { Pos }
\end{aligned}
$$

both valid ans.

Q29 $2 \log _{2} x=y$ and $\log _{2} 2 x=y+4$ find

$$
\log _{2} x^{2}=y
$$

$$
2^{y+4}=2 x
$$

$$
2^{y}=x^{2}
$$

$$
\left(2^{9}\right)\left(2^{4}\right)=2 x
$$

$$
(16) 2^{y}=2 x
$$

$$
16 x^{2}-2 x=0
$$

$$
2 x(8 x-1)=0
$$

$$
\begin{array}{rlrl}
2 x & =0 & 8 x & =1 \\
x & =0 & x & =1 / 8
\end{array}
$$



Q30

$$
\log _{6} x+\log _{6} y=1 \quad \text { show } x=\frac{6}{y}
$$

$$
\begin{gathered}
\log _{b} x y=1 \\
b^{\prime}=x y \\
\frac{b}{y}=x
\end{gathered}
$$

Solve

$$
\begin{gathered}
\log _{6} x+\log _{6} y=1 \\
5 x+y=17 \\
\frac{6 y=x}{5 x+y=17} \\
5\left(\frac{6}{y}\right)+y=17 \\
30+y^{2}=17 y \\
y^{2}-17 y+30=0 \\
(y-15) y-2)=0 \\
y=15 \quad y=2 .
\end{gathered}
$$

$$
\text { mut by } y \text {. }
$$

find $x$

$$
\begin{array}{ll}
x=\frac{6}{15} & x=\frac{6}{2} \\
x=2 / 5 & x=3 . \\
(2 / 5,15) & (3,2)
\end{array}
$$

Q31 (i)

$$
\begin{aligned}
& 4 \log _{x} 2-\log _{2} x-3=0 \\
& \log _{x} 2^{4}-\log _{2} x-3=0 \\
& \log _{x} 16-\log _{2} x-3=0
\end{aligned}
$$

Change all to log base 2

$$
\begin{gathered}
\frac{\log _{2} 16}{\log _{2} x}-\log _{2} x-3=0 \\
\frac{4}{\log _{2} x}-\log _{2} x \quad-3=0 \quad \text { Let } y-\log _{2} x \\
\frac{4}{y}-y-3=0 \quad(\text { mull by } y) \\
4-y^{2}-3 y=0 \\
0=y^{2}+3 y-4 \\
0=(y+4 x y-1) \\
\Rightarrow y^{2}=-4 \quad y=1 \\
\frac{\log _{2} x=-4 \quad \log _{2} x=1}{2^{-4}=x \quad 2=x} \begin{array}{l}
\frac{1}{2^{4}}=x \quad 2=x \\
\frac{1}{16}=x
\end{array}
\end{gathered}
$$

(ii)

$$
\begin{aligned}
& 2 \log _{1} x+1=\log _{x} 4 \\
& 2 \log _{4} x+1=\log _{x} 4
\end{aligned}
$$

Change all to base 4 .

$$
\begin{aligned}
& 2 \log _{4} x^{2}+1=\frac{\log _{4} 4}{\log _{4} x} \\
& 2 \log _{4} x^{2}+1=\frac{1}{\log _{4} x} \quad \text { Let } y-\log _{4} x . \\
& 2 y+1=\frac{1}{y} \quad(\text { malt by } y) \\
& 2 y^{2}+y=1 \\
& 2 y^{2}+y-1=0 \\
& (2 y-1)(y+1)=0 \\
& 2 y=1 \\
& y=1 / 2 \quad y=-1
\end{aligned}
$$

$\log _{1} x=1 / 2 \quad \log _{4} x=-1$

$$
\begin{array}{ll}
4^{\frac{1}{2}}=x & 4^{-1}=x \\
\sqrt{4}=x & \frac{1}{4}=x \\
2=x &
\end{array}
$$

