Exc 7.9

Q1 (i) $\log_2 4 = x$

$2^x = 4$

$2^x = 2^2$

$x = 2$

(ii) $\log_3 81 = x$

$3^x = 81$

$3^x = 3^4$

$x = 4$

(iii) $\log_{10} 1000 = x$

$10^x = 1000$

$10^x = 10^3$

$x = 3$

(iv) $\log_2 64 = x$

$2^x = 64$

$2^x = 2^6$

$x = 6$

Q2 (i) $\log_8 16 = x$

$8^x = 16$

$2^{3x} = 2^4$

$3x = 4$

$x = 4/3$

(ii) $\log_9 27 = x$

$9^x = 27$

$3^{2x} = 3^3$

$2x = 3$

$x = 3/2$

(iii) $\log_{16} 32 = x$

$16^x = 32$

$2^{4x} = 2^5$

$4x = 5$

$x = 5/4$

(iv) $\log_{\frac{1}{2}} 8 = x$

$\left(\frac{1}{2}\right)^x = 8$

$2^{-x} = 2^3$

$x = -3$

(v) $\log_{\frac{1}{3}} 81 = x$

$\left(\frac{1}{3}\right)^x = 81$

$3^{-x} = 3^4$

$x = -4$

Q3 (i) $\log_{\frac{1}{3}} 27 = x$

$\left(\frac{1}{3}\right)^x = 27$

$3^{-x} = 3^3$

$x = -3$

(ii) $\log_{\sqrt{2}} 4 = x$

$\sqrt{2}^x = 4$

$2^{\frac{1}{2}x} = 2^2$

$x = 4$

(iii) $\log_8 x = 2$

$8^2 = x$

$64 = 6$

(iv) $\log_{64} x = \frac{1}{2}$

$64^{\frac{1}{2}} = x$

$\sqrt{64} = x$

$8 = x$

Q4 (i) $\log_2 x = -1$

$2^{-1} = x$

$\frac{1}{2} = x$

(ii) $\log_3 \sqrt{27} = x$

$3^x = \sqrt{27}$

$3^x = 3^{3/2}$

$x = 3/2$

(iii) $\log_x 2 = 2$

$x^2 = 2$

$x = \sqrt{2}$

(iv) $\log_2 (0.5) = x$

$2^x = 0.5$

$2^x = 1/2$

$2^x = 2^{-1}$

$x = -1$

Q5 (i) $\log_4 2 + \log_4 32$

$\Rightarrow \log_4 (2 \times 32) = x$

$\Rightarrow \log_4 64 = x$

$4^x = 64$

$4^x = 4^3$

$x = 3$

(ii) $\log_6 9 + \log_6 8 - \log_6 2$

$\Rightarrow \log_6 \frac{(9 \times 8)}{2}$

$\Rightarrow \log_6 36 = x$

$6^x = 36$

$x = 2$.

Q5 (iii) $\log_6 4 + 2\log_6 3$

$\Rightarrow \log_6 4 + \log_6 9$

$\Rightarrow \log_6 (4 \times 9)$

$\Rightarrow \log_6 36 = x$

$6^x = 36$

$x = 2$

Q6 (i) $\log_3 2 + 2\log_3 3 - \log_3 18$

$\Rightarrow \log_3 2 + \log_3 9 - \log_3 18$

$\Rightarrow \log_3 \frac{(2 \times 9)}{18}$

$\Rightarrow \log_3 1 = 0$

(ii) $\log_8 72 - \log_8 9/8$

$\Rightarrow \log_8 \frac{72}{(9/8)} \Rightarrow \log_8 \left(72 \times \frac{8}{9}\right)$

$\Rightarrow \log_8 64 = x$

$8^x = 64$

$x = 2$.

Q7 $\log_3 5 = a$.

(i) $\log_3 15 \Rightarrow \log_3 (3 \times 5) \Rightarrow \log_3 3 + \log_3 5$

$= 1 + a$.

(ii) $\log_3 \left(\frac{5}{3}\right) \Rightarrow \log_3 5 - \log_3 3$

$= a - 1$

$\log_3 5 = a$

Q7 (iii) $\log_3\left(8\frac{1}{3}\right) \Rightarrow \log_3\left(\frac{25}{3}\right) \Rightarrow \log_3 \overset{(5\times5)}{25} - \log_3 3$

$\Rightarrow \log_3 5 + \log_3 5 - \log_3 3$

$\Rightarrow 2a - 1$

(iv) $\log_3\left(\frac{\overset{5\times5}{25}}{\underset{3\times3\times3}{27}}\right) \Rightarrow \log_3 5 + \log_3 5 - [\log_3 3 + \log_3 3 + \log_3 3]$

$\Rightarrow 2a - 3$

(or)

$\log_3 \frac{25}{27} \Rightarrow \log_3 25 - \log_3 27$

$\log_3 5^2 - \log_3 3^3$

$2\log_3 5 - 3\log_3 3$

$2a - 3(1)$

$2a - 3$

(v) $\log_3 75 \Rightarrow \log_3(25\times3) \Rightarrow 2\log_3 5 + \log_3 3$

$2a + 1$

Q8 (i) $200 = 2^x$

$\log 200 = \log 2^x$

$\log 200 = x \log 2$

$\frac{\log 200}{\log 2} = x$

(Calculator) $7.64 = x$

(ii) $5^x = 500$

$\log 5^x = \log 500$

$x \log 5 = \log 500$

$x = \frac{\log 500}{\log 5}$

$x = 3.86$

Q8 (iii) $3^{x+1} = 25$

$\log 3^{x+1} = \log 25$

$(x+1)\log 3 = \log 25$

$x+1 = \frac{\log 25}{\log 3}$

$x+1 = 2.9299$

$x = 1.93$

(iv) $5^{2x+3} = 51$

$(2x+3)\log 5 = \log 51$

$2x+3 = \frac{\log 51}{\log 5}$

$2x+3 = 2.443$

$2x = -0.557$

$x = -0.279$

Q9 $y = 2^{x-1} + 3$

(i) $2^{x-1} = y-3$

$\log 2^{x-1} = \log(y-3)$

$x-1\log_2 = \log(y-3)$

$x-1 = \frac{\log(y-3)}{\log_2}$

$x = \frac{\log(y-3)}{\log 2} + 1$

(ii) $y = 8$

$x = \frac{\log(8-3)}{\log 2} + 1$

$x = \frac{\log 5}{\log 2} + 1$

$x = 2.3219 + 1$

$x = 3.3219$

Q10 $\log_{10} x = 1+a$ $\log_{10} y = 1-a$

Show $xy = 100$

$10^{1+a} = x$ $10^{1-a} = y$

$\Rightarrow xy = (10^{1+a})(10^{1-a})$ mult $\Rightarrow$ add powers

$xy = 10^{1+a+1-a}$

$xy = 10^2$

$xy = 100.$

Q11 $P = \log_a\left(\frac{21}{4}\right)$, $q = \log_a\left(\frac{7}{3}\right)$, $r = \log_a\left(\frac{7}{2}\right)$

Show $P + q = 2r$

$\Rightarrow \log_a\left(\frac{21}{4}\right) + \log_a\left(\frac{7}{3}\right) = 2\log_a\left(\frac{7}{2}\right)$

$\Rightarrow \log_a 21 - \log_a 4 + \log_a 7 - \log_a 3 = 2\log_a\left(\frac{7}{2}\right)$

$\log_a(3\times7) - \log_a(2\times2) + \log_a 7 - \log_a 3$

$\log_a 3 + \log_a 7 - (\log_a 2 + \log_a 2) + \log_a 7 - \log_a 3$

$2\log_a 7 - 2(\log_a 2)$

$2(\log_a 7 - \log_a 2)$

$2\log_a\left(\frac{7}{2}\right)$

Hence $P + q = 2r$

Q12 $\log_a x = 4$ $\log_a y = 5$

$\Rightarrow a^4 = x$ $a^5 = y$

(i) $\log_a x^2 y$

$= \log_a x^2 + \log_a y$

$= 2\log_a x + \log_a y$

$= 2(4) + 5$

$= 8 + 5$

$= 13$

(ii) $\log_a axy$

$\log_a a + \log_a x + \log_a y$

$1 + 4 + 5$

$= 10$

(iii) $\log_a \frac{\sqrt{x}}{y}$

$\log_a x^{\frac{1}{2}} - \log_a y$

$\frac{1}{2}\log_a x - \log_a y$

$\frac{1}{2}(4) - 5$

$2 - 5 = -3$

Q13 $\log_{25} x = \frac{1}{2}\log_5 x$

$\log_{25} x = \frac{\log_5 x}{\log_5 25}$

$= \frac{\log_5 x}{\log_5 5^2}$

$= \frac{\log_5 x}{2(\log_5 5)}$ $= \frac{\log_5 x}{2} = \frac{1}{2}\log_5 x$

$[\log_5 5 = 1]$

Q14 (i) $\log_{10} 4 = 0.602$

(ii) $\log_{10} 27 = 1.43$

(iii) $\log_{10} 356 = 2.55$

(iv) $\log_{10} 5600 = 3.75$

(v) $\log_{10} 29000 = 4.46$

(vi) $\log_{10} 350{,}000 = 5.54$

(vii) $\log_{10} 3{,}870{,}000 = 6.59$

Q15 $\log_{10} x = 3.123$

$10^{3.123} = x$

$10^3 = 1000$ (min)

$10^4 = 10{,}000$ (max)

Q16 $\log_3 15 - \log_2 5$

$\frac{\log_{10} 15}{\log_{10} 3} - \frac{\log_{10} 5}{\log_{10} 2}$

$2.46497 - 2.3219 = 0.14307$

$= 0.143$

Q17 (i) $\log_{27} 81 = \frac{\log_3 81}{\log_3 27} = \frac{4}{3}$

(ii) $\log_{32} 8 = \frac{\log_2 8}{\log_2 32} = \frac{3}{5}$

Q18 $\boxed{\log_b a = \frac{1}{\log_a b}}$ NB. into Notes

Change $\log_b a$ to base a.

$$\frac{\log_a a}{\log_a b} = \frac{1}{\log_a b} \qquad \text{QED.}$$

Q19 If $x > 0$ or $x \neq 1$ show $\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_5 x} = \frac{1}{\log_{30} x}$

Using fact established in Q18.

$$\log_x 2 + \log_x 3 + \log_x 5$$
$$= \log_x (2 \times 3 \times 5)$$
$$= \log_x 30 = \frac{1}{\log_{30} x} \qquad \text{QED}$$

Q20 $\log_r P = \log_r 2 + 3\log_r q$ express P in terms of q

$\log_r P = \log_r 2 + \log_r q^3$

$\log_r P = \log_r 2q^3$

$P = 2q^3$

Q21 $\log_3 a + \log_9 a = \frac{3}{4}$ Find a

$\log_3 a + \frac{\log_3 a}{\log_3 9} = \frac{3}{4}$

$\log_3 a + \frac{\log_3 a}{\log_3 3^2} = \frac{3}{4}$

$\log_3 a + \frac{\log_3 a}{2\log_3 3} = \frac{3}{4}$

$\log_3 a + \frac{\log_3 a}{2} = \frac{3}{4}$ (mult by 2)

$2\log_3 a + \log_3 a = \frac{3}{2}$

$3\log_3 a = \frac{3}{2}$ (Divide by 3)

$\log_3 a = \frac{1}{2}$

$3^{\frac{1}{2}} = a$

$\sqrt{3} = a$.

Q22 $3\ln 41.5 - \ln 250$

$3(3.7257) - 5.5215$

$11.177 - 5.5215$

$= 5.6555$

$= 5.66$

[eg 5 + eg 6.]

Solving Logarithmic Equations

- When solving log equations, ensure that each term has the same base, if not the change of base rule must be used first.
- If no base given, the equation is true for all bases.
- If $\log_a b = \log_a c$, then $b = c$
- If $\log_a b = k$, then $b = a^k$
- Check all answers to ensure they do not produce logs of negative numbers as these are not defined.

Example 5

Solve the equation $2\log_3 x - \log_3(18 - x) = 1$.

Example 6

Solve the equation $\log_3 x + 3\log_x 3 = 4$.

Q23 $\log_2(x-2) + \log_2 x = 3$

$\log_2(x-2)(x) = 3$

$\log_2(x^2-2x) = 3$

$2^3 = x^2-2x$

$8 = x^2-2x$

$0 = x^2-2x-8$

$0 = (x-4)(x+2)$

$x = 4$ $\quad x = -2$ not valid.

Ans: $x = 4$

Q24 $\log_{10}(x^2+6) - \log_{10}(x^2-1) = 1$

$\log_{10}\left(\frac{x^2+6}{x^2-1}\right) = 1$

$10^1 = \frac{x^2+6}{x^2-1}$

$10(x^2-1) = x^2+6$

$10x^2 - 10 = x^2+6$

$9x^2 - 16 = 0$

$(3x+4)(3x-4) = 0$

$3x = -4$ $\quad 3x-4 = 0$

$x = -4/3$ $\quad 3x = 4$

$x = 4/3$

$\Rightarrow$ Valid as in eqn x is always squared $\Rightarrow$ positive.

$\therefore$ Ans: $x = \pm 4/3$

Q25 $\log 2x - \log(x-7) = \log 3$

$$\log\left(\frac{2x}{x-7}\right) = \log 3$$

$$\frac{2x}{x-7} = 3$$

$$2x = 3x - 21$$

$$21 = x$$

Q26 $\log(2x+3) + \log(x-2) = 2\log x$

$$\log(2x+3)(x-2) = \log x^2$$

$$(2x+3)(x-2) = x^2$$

$$2x^2 - 4x + 3x - 6 = x^2$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$x = 3$ $x = -2$ → Not Valid.

$\Rightarrow x = 3$ Ans

Q27 $\log_{10}(17-3x) + \log_{10} x = 1$

$$\log_{10}(17-3x)(x) = 1$$

$$10^1 = (17-3x)(x)$$

$$10 = 17x - 3x^2$$

$$3x^2 - 17x + 10 = 0$$

$$(3x-2)(x-5) = 0$$

$x = 2/3$ $x = 5$

Q28 $\log_{10}(x^2-4x-11)=0$

$10^0 = x^2-4x-11$

$1 = x^2-4x-11$

$0 = x^2-4x-12$

$0 = (x-6)(x+2)$

$x=6 \quad x=-2 \qquad (-2)^2-4(-2)-11 = 1$ Pos

both Valid ans.

Q29 $2\log_2 x = y$ and $\log_2 2x = y+4$ find x

$\log_2 x^2 = y$

$2^y = x^2$

$2^{y+4} = 2x$

$(2^y)(2^4) = 2x$

$(16)2^y = 2x$

$16x^2-2x=0$

$2x(8x-1)=0$

$2x=0 \quad 8x=1$

$x=0 \quad x=1/8$

not Valid

Q30 $\log_6 x + \log_6 y = 1$ Show $x = \frac{6}{y}$

$\log_6 xy = 1$

$6^1 = xy$

$\frac{6}{y} = x$

Solve $\log_6 x + \log_6 y = 1$

$5x + y = 17$

$\frac{6}{y} = x$

$5x + y = 17$

$5\left(\frac{6}{y}\right) + y = 17$ mult by y.

$30 + y^2 = 17y$

$y^2 - 17y + 30 = 0$

$(y - 15)(y - 2) = 0$

$y = 15 \quad y = 2$.

Find x $x = \frac{6}{y}$

$x = \frac{6}{15} \qquad x = \frac{6}{2}$

$x = 2/5 \qquad x = 3$.

$(\frac{2}{5}, 15) \qquad (3, 2)$

Q31 (i) $4\log_x 2 - \log_2 x - 3 = 0$.

$\log_x 2^4 - \log_2 x - 3 = 0$.

$\log_x 16 - \log_2 x - 3 = 0$

Change all to log base 2.

$\frac{\log_2 16}{\log_2 x} - \log_2 x - 3 = 0$

$\frac{4}{\log_2 x} - \log_2 x - 3 = 0$ let $y = \log_2 x$

$\frac{4}{y} - y - 3 = 0$ (mult by y)

$4 - y^2 - 3y = 0$

$0 = y^2 + 3y - 4$

$0 = (y+4)(y-1)$

$y = -4$ $y = 1$

$\Rightarrow$ $\log_2 x = -4$ $\log_2 x = 1$

$2^{-4} = x$ $2^1 = x$

$\frac{1}{2^4} = x$ $2 = x$

$\frac{1}{16} = x$

(ii) $2\log_4 x + 1 = \log_x 4$

$2\log_4 x^{\frac{1}{2}} + 1 = \log_x 4$

Change all to base 4.

$2\log_4 x^{\frac{1}{2}} + 1 = \frac{\log_4 4}{\log_4 x}$

$2\log_4 x^{\frac{1}{2}} + 1 = \frac{1}{\log_4 x}$ Let $y = \log_4 x$.

$2y^{\frac{1}{2}} + 1 = \frac{1}{y}$ (mult by y)

$2y^2 + y = 1$

$2y^2 + y - 1 = 0$

$(2y - 1)(y + 1) = 0$

$2y = 1$

$y = 1/2$ $y = -1$

$\log_4 x = 1/2$ $\log_4 x = -1$

$4^{\frac{1}{2}} = x$ $4^{-1} = x$

$\sqrt{4} = x$ $\frac{1}{4} = x$

$2 = x$