Revision Exercise (Core)

Q1

(i) $T_n = 3n + 4$; $\quad 7, 10, 13, 16.$

(ii) $T_n = 6n - 1$; $\quad 5, 11, 17, 23$

(iii) $T_n = 2^{n-1}$; $\quad 1, 2, 4, 8$

(iv) $t_n = (n+3)(n+4)$; $\quad 20, 30, 42, 56.$

(v) $T_n = n^3 + 1$; $\quad 2, 9, 28, 65.$

Q2 Arith: $T_n = a + (n-1)d.$

$T_3 = 71$

$\boxed{a + 2d = 71}$

$T_7 = 55$

$\boxed{a + 6d = 55}$

$$
\begin{aligned}
\cancel{a} + 6d &= 55 \\
-\cancel{a} - 2d &= -71 \\
\hline
4d &= -16 \\
d &= -4
\end{aligned}
$$

Find a:

$$
\begin{aligned}
a + 2(-4) &= 71 \\
a - 8 &= 71 \\
a &= 79.
\end{aligned}
$$

$$
\begin{aligned}
T_n &= 79 + (n-1)(-4) \\
&= 79 - 4n + 4 \\
&= 83 - 4n
\end{aligned}
$$

Q3 Geometric: $T_n = ar^{n-1}$ $\quad S_\infty = \frac{a}{1-r}$

$T_1 = 12$ $\quad S_\infty = 36.$

$a = 12$ $\quad \frac{12}{1-r} = 36$

$$12/36 = 1-r$$

$$1/3 = 1-r$$

$$r = 1 - 1/3$$

$$r = 2/3$$

Q4 (i) $-2, 4, -8 \ldots\ldots \quad a = -2 \quad r = \frac{4}{-2} = -2.$

$T_n = ar^{n-1} \qquad T_n = (-2)(-2)^{n-1}$

(ii) $1, \frac{1}{2}, \frac{1}{4} \ldots\ldots \quad a = 1 \quad r = \frac{1/2}{1} = \frac{1}{2}.$

$T_n = ar^{n-1} \qquad T_n = \left(\frac{1}{2}\right)^{n-1} = \frac{1}{2^{n-1}} = 2^{1-n}.$

(iii) $2, -6, 18 \ldots\ldots \quad a = 2 \quad r = \frac{-6}{2} = -3$

$T_n = ar^{n-1} \qquad T_n = (2)(-3)^{n-1}$

Q5 (i) N° of Sticks: 12, 20, 28. diff = 8

$$T_n = 8n + 4$$

(ii)

$$8n + 4 = 2006$$
$$8n = 2002$$
$$n = \frac{2002}{8}$$
$$n = 250.25$$

$\Rightarrow$ Max N° is 250

Q6 Geometric $\Rightarrow$ $T_n = ar^{n-1}$

(i) $T_2 = 21$ $T_3 = -63$.

$ar^1 = 21$ $ar^2 = -63$

$\frac{\cancel{a}r^2}{\cancel{a}r^1} = \frac{-63}{21}$ $\Rightarrow$ $r = -3$.

(ii) find a: $ar = 21$

$a(-3) = 21$

$a = -7$

Q7 2000 invested at 2.5%

$\Rightarrow$ Yr 1 $= 2000 + (2000)(0.025)$

$= 2000(1 + 0.025)^1$

$= 2000(1.025)^1$

Yr 2 $= 2000(1.025) + 2000(1.025)(0.025)$

$= 2000(1.025)[1 + 0.025]$

$= 2000(1.025)(1.025)$

$= 2000(1.025)^2$

$\Rightarrow$ After 5 Yrs $= 2000(1.025)^5$

Q8 $1 + 2 + 3 + 4 + \dots 200$

$$S_n = \frac{n}{2}\{2a + (n-1)d\} \qquad a = 1 \quad d = 1 \quad n = 200$$

$$S_{200} = \frac{200}{2}\{2(1) + 199(1)\}$$

$$= 100(2 + 199)$$

$$= 100(201)$$

$$= 20{,}100$$

Q9 Arith. $T_n = a + (n-1)d$.

$$T_5 = 2T_2$$

$$a + 4d = 2(a + d)$$
$$a + 4d = 2a + 2d$$
$$0 = 2a - 4d \quad (\div 2)$$
$$\boxed{0 = a - 2d}$$

$$T_5 - T_2 = 9.$$

$$(a + 4d) - (a + d) = 9$$
$$3d = 9$$
$$\boxed{d = 3} \quad \Rightarrow \quad 0 = a - 2(3)$$
$$\boxed{6 = a}$$

$$\Rightarrow T_n = 6 + (n-1)3 = 6 + 3n - 3 = \underline{\underline{3n + 3}}$$

$$\Rightarrow T_n = 6 + (n-1)3 = 6 + 3n - 3 = \underline{\underline{3n+3}}$$

$$S_n = \frac{n}{2}\{2a + (n-1)d\} \qquad a = 6,\ d = 3 \quad n = 10.$$

$$S_{10} = \frac{10}{2}\{2(6) + (9)(3)\}$$

$$= 5(12 + 27)$$

$$= 5(39)$$

$$= 195.$$

Q10 $\sum_{r=3}^{16} (2r+1)$

$\Rightarrow \quad 2(3)+1 = 7$
$2(4)+1 = 9$
$2(5)+1 = 11$
$\vdots$
$2(16)+1 = 33.$

$T_n = a+(n-1)d.$
$T_n = 7+(n-1)(2) = 7+2n-2 = 2n+5.$

$2n+5 = 33$
$2n = 28$
$n = 14.$ Terms

$S_n = \frac{n}{2}\{2a+(n-1)d\} \qquad a=7 \quad d=2 \quad n=14$
$S_{14} = \frac{14}{2}\left(2(7)+(13)(2)\right)$
$= 7(14+26)$
$= 7(40)$
$= 280.$

Revision Exercises (Advanced)

Q1

(i) $$2000, \overset{\times 3/5}{1200}, \overset{(\times 3/5)^2 \dots}{720}$$

$$a = 2000, \quad r = 3/5$$

$$T_n = ar^n \qquad T_{10} = 2000(3/5)^{10} = 20 \text{ lumens}$$

(ii) $$T_n = 2000(3/5)^n$$

(iii) $$\frac{1}{10} \text{ of } 2000 = 200.$$

$$2000(3/5)^n = 200$$

$$(3/5)^n = 0 \cdot 1$$

$$n \log 3/5 = \log 0 \cdot 1$$

$$n = \frac{\log 0 \cdot 1}{\log(3/5)}$$

$$n = 4 \cdot 51$$

$\Rightarrow$ after 5^{th} Mirror.

Q2 $\quad A = P(1+i)^t$

(i) Double initial investment $= 2P$.

$$2\cancel{P} = \cancel{P}(1+i)^t$$

$$2 = (1+i)^t$$

$\Rightarrow$ t for doubling depends only on i.

Solve for t.

$$\log 2 = t \log(1+i)$$

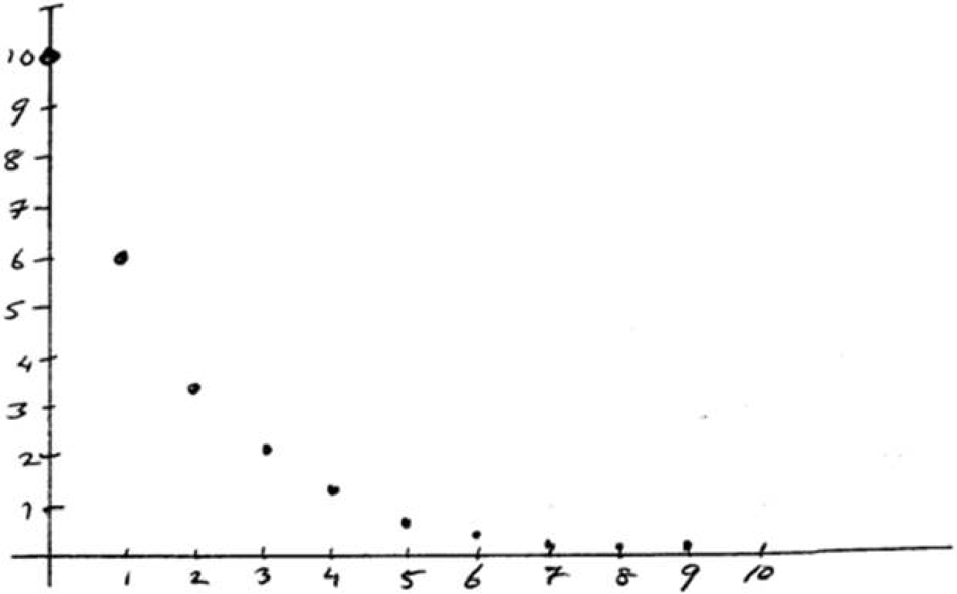
$$\frac{\log 2}{\log(1+i)} = t$$

(ii) $i = 2\% \Rightarrow t = \dfrac{\log 2}{\log(1+0.02)} = 35$ yrs

$i = 5\% \Rightarrow t = \dfrac{\log 2}{\log(1+0.05)} = 14.2$ yrs

$i = 10\% \Rightarrow t = \dfrac{\log 2}{\log(1+0.1)} = 7.3$ yrs.

Q3 10, 6, 3·6. $r = \frac{6}{10} = 0·6$



10, 6, 3·6, 2·16, 1·3, 0·78, 0·47, 0·28, 0·17, 0·1

$0 \cdot 17 \ , \ 0 \cdot 1$

(i) Total Distance $= 10 + 2(6 + 3 \cdot 6 + 2 \cdot 16 + \ldots)$

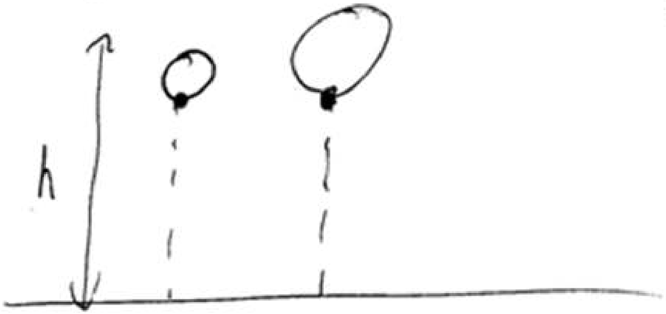
(ii) An Infinite geometric series

(iii) $S_\infty = \frac{a}{1-r}$ $\quad a = 6 \quad r = 0 \cdot 6$

$S_\infty = \frac{6}{1 - 0 \cdot 6} = 15$

$\Rightarrow$ Dis $= 10 + 2(15) = 40m.$

(iv) If a point at the base of the ball is taken to calculate the height Then the size of the ball has no effect.



Q4 (i) 3, 6, 12, 24, 48,

$a = 3 \quad r = 6/3 = 2.$

$T_n = ar^{n-1}$
$= 3.(2^{n-1})$

(ii) $T_n > 1,000,000$

$3(2^{n-1}) > 1,000,000$

$2^{n-1} > \frac{1,000,000}{3}$

$n-1 \log 2 > \log 333,333{\cdot}3$

$n-1 > \frac{\log 333,333{\cdot}3}{\log 2}$

$n-1 > 18{\cdot}35$

$n > 18{\cdot}35 + 1$
$n > 19{\cdot}35$

$\Rightarrow$ 20^{th} term $> 1,000,000$

Q5 $1, \ 2, \ 4, \ 8, \ 16, \ \ldots$

$a = 1 \qquad r = 2/1 = 2.$

$T_n = ar^{n-1}$
$= 1(2^{n-1})$

(i) $T_{32} = 1(2^{32-1}) = 2^{31} = 2147483648$ cent.
$= €21,474,836.$

(ii) $T_{64} = 2^{63} = 9 \cdot 22 \times 10^{18}$ cent
$= €9 \cdot 22 \times 10^{16}$

Q6 Arithmetic

Let Sequence be $a, \; a+d, \; a+2d, \; \ldots$

$$a+(a+d)+(a+2d) = 33$$
$$3a+3d = 33$$
$$\boxed{a+d = 11}$$

$$(a)(a+d)(a+2d) = 935$$
$$(a^2+ad)(a+2d) = 935$$
$$a^3+2a^2d+a^2d+2ad^2 = 935$$
$$\boxed{a^3+3a^2d+2ad^2 = 935}$$

$d = \underline{\underline{11-a}}$

$$a^3+3a^2(11-a)+2a(11-a)^2 = 935$$
$$a^3+33a^2-3a^3+2a(121-22a+a^2) = 935$$
$$\cancel{a^3}+33a^2-\cancel{3a^3}+242a-44a^2+\cancel{2a^3} = 935$$
$$-11a^2+242a-935 = 0$$

$$11a^2 - 242a + 935 = 0 \quad (\div 11)$$
$$a^2 - 22a + 85 = 0$$
$$(a - 17)(a - 5) = 0$$
$$a = 17 \qquad a = 5$$

<u>find</u> d $\qquad d = 11 - a$.

at $a = 17$: $d = 11 - 17$
$d = -6$

$\Rightarrow$ Seq: $17, 11, 5, \ldots$

at $a = 5$: $d = 11 - 5$
$d = 6$

$\Rightarrow$ Seq: $5, 11, 17, \ldots$

OR

* Q6 Let Seq be $a-d,\ a,\ a+d$.

$$(a-d)+(a)+(a+d)=33$$
$$3a=33$$
$$a=11. \quad [\text{middle Term}]$$

$$(a-d)(a)(a+d)=935.$$
but $a=11$
$$(11-d)(11)(11+d)=935 \quad (\div 11)$$
$$(11-d)(11+d)=85$$
$$121-d^2=85$$
$$d^2=36$$
$$d=\pm 6.$$

Seq when $d=6$: 5, 11, 17

Seq when $d=-6$: 17, 11, 5.

Q7 Car new = 30,000 ↓ 13% per Year. ($\Rightarrow \cdot 13$)

(i) Yr 1 : $30{,}000 - (\cdot 13)(30{,}000)$

$= 30{,}000\,(1 - 0\cdot 13)$

Yr 2 : $30000\,(1 - 0\cdot 13) - [(30{,}000)(1 - 0\cdot 13)](0\cdot 13)$

$= 30{,}000(1 - 0\cdot 13)[1 - 0\cdot 13]$

$= 30{,}000\,(1 - 0\cdot 13)^2$

$\boxed{\text{After 'a' years : } 30{,}000(1 - 0\cdot 13)^a}$

(ii) 5 yrs $\Rightarrow$ a = 5.

$30{,}000\,(1 - 0\cdot 13)^5 = €14{,}953$

(iii) less than 6000

$$30{,}000(1-0.13)^a < 6000$$

$$(1-0.13)^a < \frac{6000}{30{,}000}$$

$$(0.87)^a < 0.2$$

$$a \log(0.87) < \log(0.2)$$

$$a < \frac{\log(0.2)}{\log(0.87)}$$

$$a < 11.56$$

$\Rightarrow$ in the 12th year.

Q8 $\quad T_n = 3\left(\frac{2}{3}\right)^n - 1$

(i) $T_1 = 3\left(\frac{2}{3}\right)^1 - 1 = 2 - 1 = 1$

$T_2 = 3\left(\frac{2}{3}\right)^2 - 1 = 3\left(\frac{4}{9}\right) - 1 = \frac{4}{3} - 1 = \frac{1}{3}$

$T_3 = 3\left(\frac{2}{3}\right)^3 - 1 = 3\left(\frac{8}{27}\right) - 1 = \frac{8}{9} - 1 = -\frac{1}{9}$

(ii) Show that $T_{n+1} = 2\left(\frac{2}{3}\right)^n - 1$

$$T_{n+1} = 3\left(\frac{2}{3}\right)^{n+1} - 1$$

$$= \cancel{3}\left(\frac{2}{3}\right)^n\left(\frac{2}{\cancel{3}}\right) - 1$$

$$= 2\left(\frac{2}{3}\right)^n - 1 \qquad \text{QED.}$$

(iii) $3T_{n+1} - 2T_n = k \qquad$ find k.

$$3\left[2\left(\frac{2}{3}\right)^n - 1\right] - 2\left[3\left(\frac{2}{3}\right)^n - 1\right] = k$$

$$\cancel{6\left(\frac{2}{3}\right)^n} - 3 - \cancel{6\left(\frac{2}{3}\right)^n} + 2 = k$$

$$-3 + 2 = k$$

$$-1 = k$$

Q8 (iv) Show $\sum_{1}^{15}\left[3\left(\frac{2}{3}\right)^{n}-1\right] = -9.014$.

$$\sum_{1}^{15}\left[3\left(\frac{2}{3}\right)^{n}-1\right] = \left[\sum_{1}^{15} 3\left(\frac{2}{3}\right)^{n}\right] - 15.$$

$$\begin{array}{ccc} T_1 & T_2 & T_3 \\ 2, & \frac{4}{3}, & \frac{8}{9} \end{array} \quad \ldots \qquad \Rightarrow a = 2 \quad r = 2/3$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{2\left(1-2/3^n\right)}{1-2/3} = 6\left(1-2/3^n\right)$$

$$S_{15} = 6\left(1-(2/3)^{15}\right) = 5.986298$$

$$\left[\sum_{1}^{15} 3\left(\frac{2}{3}\right)^{n}\right] - 15$$

$$5.986298 - 15 = -9.014.$$ QED

Q9 (i) Show $T_n = S_n - S_{n-1}$

$$S_n = \cancel{T_1} + \cancel{T_2} + \cancel{T_3} + \cdots \cancel{T_{n-1}} + T_n$$

$$\ominus \quad S_{n-1} = \cancel{T_1} + \cancel{T_2} + \cancel{T_3} + \cdots \cancel{T_{n-1}}$$

$$S_n - S_{n-1} = T_n.$$

(ii) $S_n = 3n^2 + n$. find T_n.

$$S_{n-1} = 3(n-1)^2 + (n-1)$$

$$\begin{aligned}
T_n &= S_n - S_{n-1} \\
&= (3n^2+n) - (3(n-1)^2 + (n-1)) \\
&= 3n^2 + n - [3(n^2-2n+1) + (n-1)] \\
&= 3n^2 + n - [3n^2 - 6n + 3 + n - 1] \\
&= \cancel{3n^2} + \cancel{n} - \cancel{3n^2} + 6n - 3 - \cancel{n} + 1 \\
T_n &= 6n - 2
\end{aligned}$$

$$= 3n^2 + n - [3n^2 - 6n + 3 + n - 1]$$

$$= \cancel{3n^2} + \cancel{n} - \cancel{3n^2} + 6n - 3 - \cancel{n} + 1$$

$$\underline{\underline{T_n = 6n - 2}}$$

(iii) $$\sum_{r=1}^{n} (T_r)^2 = T_1^2 + T_2^2 + T_3^2 + \cdots T_n^2$$

$$= (6-2)^2 + (6(2)-2)^2 + (6(3)-2)^2 \cdots (6(n)-2)^2$$

$$= \sum (6n-2)^2$$

$$= \sum (36n^2 - 24n + 4)$$

$$= 36\sum_{n=1}^{n} n^2 - 24\sum_{n=1}^{n} n + \sum_{n=1}^{n} 4.$$

but $\sum n = \frac{n}{2}(n+1)$, $\sum_{n=1}^{n} 4 = 4n$, $\sum_{n=1}^{n} n^2 = \frac{n}{6}(2n+1)(n+1)$

$$\sum (T_r)^2 = 36\left[\frac{n}{6}(2n+1)(n+1)\right] - 24\left[\frac{n}{2}(n+1)\right] + 4n$$

$$= n(n+1)\left[12n+6-12\right] + 4n$$

$$= n(n+1)(12n-6) + 4n$$

$$= n\left[(n+1)(12n-6) + 4\right]$$

$$= n\left[12n^2 + 6n - 2\right]$$

$$= 2n\left[6n^2 + 3n - 1\right].$$

Q10 $\log_4 x$ in terms of $\log_2 x$

$$\log_4 x = \frac{\log_2 x}{\log_2 4}$$

let $y = (\log_2 4)$

$$\Downarrow$$

$$2^y = 4$$

$$2^y = 2^2$$

$$y = 2.$$

$$\Rightarrow \frac{\log_2 x}{2.}$$

$$\log_{16} x = \frac{\log_2 x}{\log_2 16} = \frac{\log_2 x}{4.}$$

$$\therefore \quad \log_2 x, \quad \log_4 x, \quad \log_{16} x$$

$$= \log_2 x, \quad \frac{\log_2 x}{2}, \quad \frac{\log_2 x}{4}$$

$$\therefore \quad \log_2 x \,,\quad \log_4 x \,,\quad \log_{16} x$$

$$= \log_2 x \,,\quad \frac{\log_2 x}{2} \,,\quad \frac{\log_2 x}{4}$$

$$\Rightarrow a = \log_2 x \qquad r = \tfrac{1}{2}.$$

$$\lim_{n\to\infty} S_n = \frac{a}{1-r} \quad \Rightarrow S_\infty = \frac{\log_2 x}{1-\frac{1}{2}}$$

$$= \frac{\log_2 x}{\frac{1}{2}} \quad = \quad 2\log_2 x$$

$$\therefore \quad k\log_2 x = 2\log_2 x$$

$$\Rightarrow \quad k = 2.$$

Revision Exercises [Extended-Response Questions]

Q1 $T_n = an^3 + bn^2 + cn + d.$

(i)

T_1	T_2	T_3	T_4	T_5	
$a+b+c+d$	$8a+4b+2c+d$	$27a+9b+3c+d$	$64a+16b+4c+d$	$125a+25b+5c+d$	
$7a+3b+c$	$19a+5b+c$	$37a+7b+c$	$61a+9b+c$		1st Diff
$12a+2b$	$18a+2b$	$24a+2b$			2nd Diff
$6a$	$6a$.				3rd Diff

(ii) The 3rd diff for all cubic seq is always $6a$.

(iii) The 2nd diff for all ~~others~~ Quadratic seq is always $2a$.

The 1st diff $^{T_2 - T_1}$ for all Quadratic seq is always $3a+b$.

$T_n = an^2 + bn + c$.

T_1	T_2	T_3	T_4	
$a+b+c$	$4a+2b+c$	$9a+3b+c$	$16a+4b+c$	
$3a+b$	$5a+b$	$7a+b$		1st diff
$2a$	$2a$			2nd diff

(iv)

T_1	T_2	T_3	T_4	
5	12	25	44	
7	13	19		1st diff
6	6			2nd diff

(v) $2a = 6 \Rightarrow a = 3$.

$3a + b = 7$

$3(3) + b = 7$

$b = -2$.

$T_n = an^2 + bn + c$.

$\Rightarrow T_n = 3n^2 - 2n + c$

But $T_1 = 5$

$\Rightarrow 5 = 3(1)^2 - 2(1) + c$

$5 = 1 + c$

$4 = c$.

$\boxed{\therefore T_n = 3n^2 - 2n + 4}$

(vi) $T_{20} = 3(20)^2 - 2(20) + 4 = 1164$.

Q2 Ball dropped from 40m.
Tenth bounce = 1m.

(i) $H = 40m$

after 1 bounce = $40.r^1$
after 2 bounces = $40.r^2$
after 3 bounces = $40.r^3$

after n bounces = $40.r^n$ where r = the fraction how far bounces back up.

(ii)

$T_{10} = 1$
$40r^{10} = 1m.$

$r^{10} = \frac{1}{40}$

$10 \log r = \log \frac{1}{40}$

$\log r = \frac{\log \frac{1}{40}}{10}$

$\log r = -0.1602$

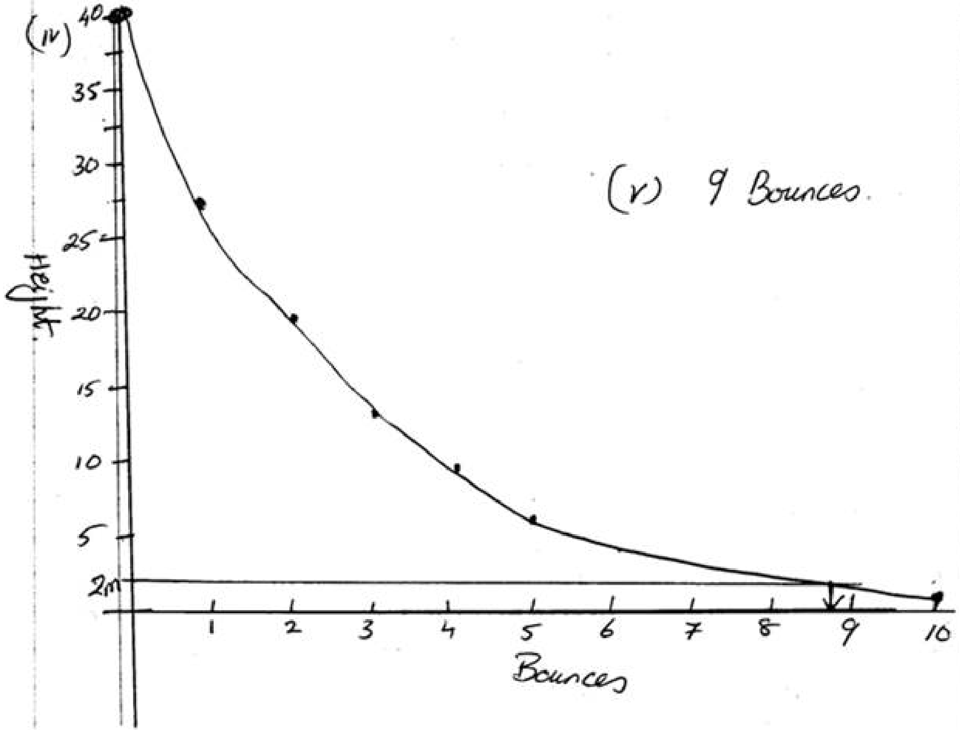
$r = 10^{-0.1602}$

$r = 0.6915 = 69\%$.

(iii) $T_1 = 40(0.69)^1 = 27.6$
$T_2 = 40(0.69)^2 = 19.044$
$T_3 = 40(0.69)^3 = 13.14$
$T_4 = 40(0.69)^4 = 9.067$
$T_5 = 40(0.69)^5 = 6.256$

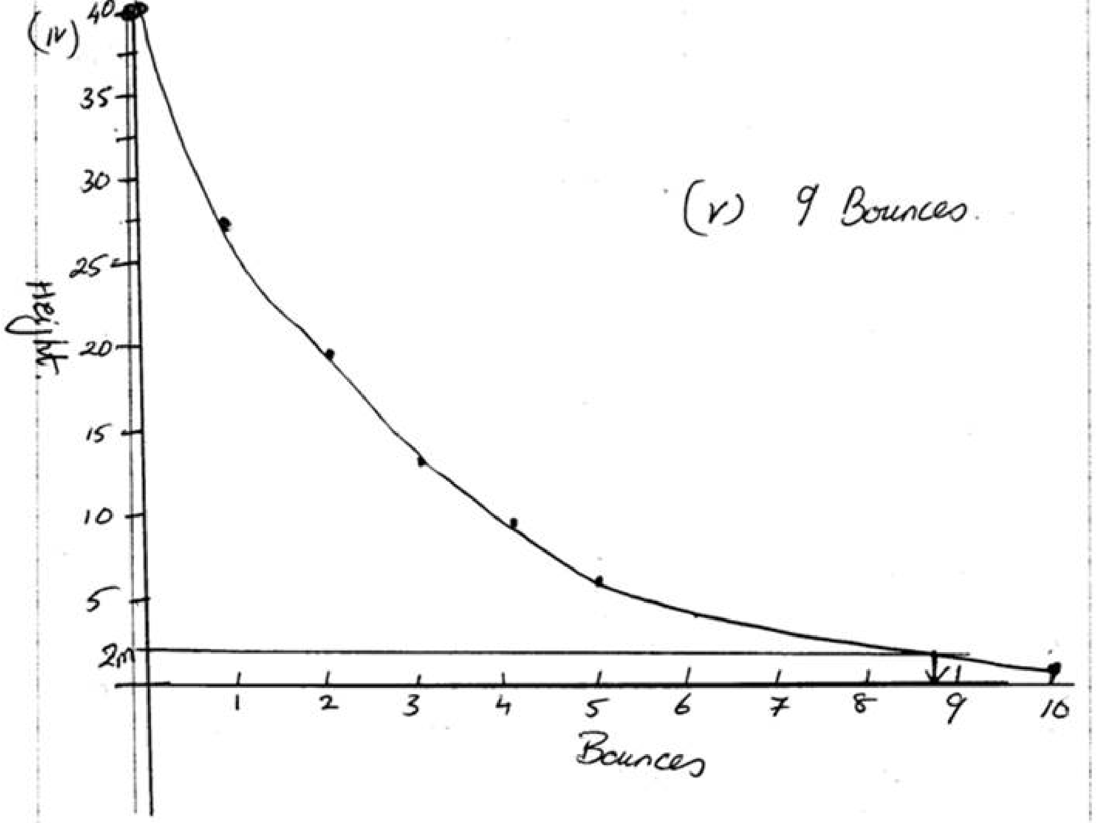
Bounce	1st	2nd	3rd	4th	5th
Height	27.6m	19.04m	13.14m	9.07m	6.26m.

(iv)



(v) 9 Bounces.

(iv)



(v) 9 Bounces.

(vi) $40(0{\cdot}69)^n < 2$

$0{\cdot}69^n < 0{\cdot}05$

$n \log 0{\cdot}69 < \log 0.05$

$n < \dfrac{\log 0.05}{\log 0{\cdot}69}$

$n < 8{\cdot}073$

$\Rightarrow$ 9 bounces

(vii) Real life conditions vary. Ball loses energy eventually stops.

Q3 Scheme 1: 20, 22, 24, 26, ...

$a = 20 \quad d = 2$

$$T_n = 20 + (n-1)2$$
$$= 20 + 2n - 2$$
$$= 18 + 2n.$$

Total amount of Money! $\Rightarrow S_n$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$= \frac{n}{2}(40 + 2n - 2)$$
$$= \frac{n}{2}(38 + 2n)$$
$$= n(19 + n)$$
$$= n^2 + 19n.$$

$\underline{\text{Scheme 2}}$: $20, \quad 20\left(\frac{21}{20}\right), \quad 20\left(\frac{21}{20}\right)^2, \quad 20\left(\frac{21}{20}\right)^3 \cdots$

$$a = 20 \qquad r = \frac{21}{20}$$

$$T_n = ar^{n-1}$$
$$= 20\left(\frac{21}{20}\right)^{n-1}.$$

$\underline{\text{Total}}$ amt of Money $\Rightarrow S_n$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{20\left(1-\left(\frac{21}{20}\right)^n\right)}{1-\frac{21}{20}}$$

$$\frac{20\left(1-\left(\frac{21}{20}\right)^n\right)}{\frac{-1}{20}} \times \frac{-20}{1} = -400\left(1-\left(\frac{21}{20}\right)^n\right)$$

$$= 400\left(\left(\frac{21}{20}\right)^n - 1\right)$$

(ii) 36 weeks.

Scheme 1 : $S_{36} = (36)^2 + 19(36) = €1980$

Scheme 2 : $S_{36} = 400\left(\left(\frac{21}{20}\right)^{36} - 1\right) = €1916.73$

$\Rightarrow$ Scheme 1 is Better.

(iii) Spend €1·50 per school day $\Rightarrow 1.50 \times 5 = 7.50$
Spent each wk.

	Wk 1	Wk 2	Wk 3	
saves	12·50	14·50	16·50	- - - -

$a = 12.5 \qquad d = 2.$

$\boxed{\text{needs } €400}$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$400 = \frac{n}{2}\{2(12.5) + (n-1)2\}$$

$$800 = n(25 + 2n - 2)$$
$$800 = n(23 + 2n)$$
$$800 = 23n + 2n^2$$
$$2n^2 + 23n - 800 = 0$$

$$n = \frac{-23 \pm \sqrt{(23)^2 - 4(2)(-800)}}{2(2)}$$

$$= \frac{-23 \pm \sqrt{6929}}{4}$$

$$15.06 \qquad \cancel{-26.56}$$

$\Rightarrow$ Can buy it in Week 16.

Q4 2010 : 160 litres. 15% Lost in 1 Yr.

(i) 15% lost $\Rightarrow$ 85% left

$160 \times .85 = 136$ litres left.

(ii) $2010 \rightarrow 2020 \Rightarrow 20$ yrs.

End Yr 1 $= 160(.85)^1$

End Yr 2 $= 160(.85)^2$

$\Rightarrow$ End Yr 10 $= 160(.85)^{10}$

$= 31.4999$ litres

$= 31.5$ litres.

(iii)

Last based left End 1 Yr $= 160(.85) = 136$ lit

(iii)

last barrel left for 1 yr $= 160(\cdot 85) = 136$ lit.
2^{nd} last barrel left for 2 yrs $= 160(\cdot 85)^2 = 115\cdot 6$.
etc.

$\Rightarrow$ after 20 yrs. Total liquid.

$$= 160(\cdot 85) + 160(\cdot 85)^2 + 160(\cdot 85)^3 + \cdots 160(\cdot 85)^{20}.$$

$a = 136 \quad r = 0\cdot 85.$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{136\left(1-(0\cdot 85)^{20}\right)}{1-0\cdot 85}$$

$= 871\cdot 52$ litres

$= 872$ litres.

Q5 Bought for €15,000 in 2005
Decreases by 20% each yr.

(i) 2005 → 2007 = 2 yrs
Decrease by 20% ⇒ 80% of Value left.

$$15000(\cdot 80)^2 = €9600$$

(ii) $15000(\cdot 80)^n < 500$

$$\cdot 80^n < \frac{500}{15000}$$

$$\cdot 80^n < \frac{1}{30}$$

$$n \log \cdot 8 < \log \tfrac{1}{30}$$

$$n < \frac{\log 1/30}{\log \cdot 8}$$

$n < 15.24$ yrs.

⇒ 2005 + 15 yrs = 2020.

$$n < \frac{\log \cdot 3 \ldots}{\log \cdot 8}$$

$n < 15.24$ yrs.

$\Rightarrow 2005 + 15$ yrs $= 2020$.

(iii) €1000 each yr, 5% Int.

$\Rightarrow$ Savings $= 1000(1.05) + 1000(1.05)^2 + \cdots 1000(1.05)^{15}$

$a = 1000(1.05) \quad r = 1.05$.

$$S_{15} = \frac{1000(1.05)\left[1 - (1.05)^{15}\right]}{1 - 1.05}$$

$= 22657.49$

$=$ €22657

(iv) Saved $= €22657$

Inflation $= r\%$

Cost of new machine in 15 yrs time $= 15000(1.05)^{15}$

$$15000(1.0r)^{15} = 22657$$

$$(1.0r)^{15} = \frac{22657}{15000}$$

$$(1.0r)^{15} = 1.51$$

$$1.0r = (1.51)^{1/15}$$

$$1.0r = 1.028$$

$$r = 0.028$$

$$r\% = 2.8\%$$