

Ex 2.5

Q1 $y = x^3 + 2x^2$
 $\frac{dy}{dx} = 3x^2 + 4x$
 $\frac{d^2y}{dx^2} = 6x + 4$

Q5 $y = 3x + \frac{1}{x} + 4$
 $= 3x + x^{-1} + 4$
 $\frac{dy}{dx} = 3 - x^{-2}$
 $\frac{d^2y}{dx^2} = +2x^{-3} = \frac{2}{x^3}$

Q8 $y = (3x-2)^3$
 $\frac{dy}{dx} = 3(3x-2)^2(3)$
 $= 9(3x-2)^2$
 $\frac{d^2y}{dx^2} = 18(3x-2)(3)$
 $= 54(3x-2)$

Q11 $f(x) = 3x + \frac{4}{x}$ show $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = 0$

$$f(x) = 3x + 4x^{-1}$$

$$f'(x) = 3 - 4x^{-2}$$

$$f''(x) = 8x^{-3}$$

$$x^2 \left(\frac{8}{x^3} \right) + x \left(3 - \frac{4}{x^2} \right) - \left(3x + \frac{4}{x} \right)$$

$$\frac{8}{x} + 3x - \frac{4}{x} - 3x - \frac{4}{x}$$

$$\frac{8 - 4 - 4}{x} = \frac{0}{x} = 0 \quad \text{QED}$$

Q13 $y = x^4$ show $\frac{4x^4}{3} \left(\frac{d^2 y}{dx^2} \right) - \left(\frac{dy}{dx} \right)^2 = 0$

$$\frac{dy}{dx} = 4x^3$$

$$\frac{d^2 y}{dx^2} = 12x^2$$

$$\frac{4x^4}{3} (12x^2) - (4x^3)^2 = 0$$

$$16x^6 - 16x^6 = 0$$

$$0 = 0 \quad \text{QED}$$

Q14

$$y = \frac{1}{\sqrt{x}}$$

Show $2x\left(\frac{d^2y}{dx^2}\right) + 3\frac{dy}{dx} = 0$

$$y = x^{-1/2}$$

$$\frac{dy}{dx} = -\frac{1}{2}x^{-3/2}$$

$$\frac{d^2y}{dx^2} = \frac{3}{4}x^{-5/2}$$

$$2x\left(\frac{3}{4\sqrt{x^5}}\right) + 3\left(\frac{-1}{2\sqrt{x^3}}\right)$$

$$= \frac{6x}{4x^{5/2}} - \frac{3}{2x^{3/2}}$$

$$= \frac{3}{2}x^{-3/2} - \frac{3}{2}x^{-3/2} = 0 \quad \text{Q.E.D.}$$